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Time-varying joint distribution through copulas

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ABSTRACT

The analysis of temporal dependence in multivariate time series is considered. The dependence structure between the marginal series is modelled through the use of copulas which, unlike the correlation matrix, give a complete description of the joint distribution. The parameters of the copula function vary through time, following certain evolution equations depending on their previous values and the historical data. The marginal time series follow standard univariate GARCH models. Full Bayesian inference is developed where the whole set of model parameters is estimated simultaneously. This represents an essential difference from previous approaches in the literature where the marginal and the copula parameters are estimated separately in two consecutive steps. Moreover, a Bayesian procedure is proposed for the estimation of several measures of risk, such as the variance, Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) of a portfolio of assets, providing point estimates and predictive intervals. The proposed copula model enables to capture the dependence structure between the individual assets which strongly influences these risk measures. Finally, the problem of optimal portfolio selection based on the estimation of mean–variance, mean–VaR and mean–CVaR efficient frontiers is also addressed. The proposed approach is illustrated with simulated and real financial time series.

1. Introduction

There has been much work on the extension to the multivariate case of the successful univariate autoregressive conditional heteroscedasticity (ARCH) and the generalized ARCH (GARCH) models in order to describe temporal dependence in financial data, see e.g. Bauwens et al. (2006) for a survey on multivariate GARCH models. The usual assumption in most cases is that the conditional joint distribution of the returns follows a multivariate normal or multivariate t-distribution. However, it is well known that these elliptical distribution models require a very strong symmetry of the data and might not be appropriate in many circumstances.

A recent alternative approach to the study of dependence in financial time series is the use of copulas, see e.g. Nelsen (2006). The main advantage of this approach is that the individual marginal densities of the returns can be defined separately from their dependence structure. Then, the models for the marginal time series can be firstly specified using the required univariate characteristics and then, the dependence between the returns can be completely modeled by selecting an appropriate copula function. Using this approach, many non-elliptical and flexible multivariate distributions can be obtained. In this context, most researchers have considered copula GARCH models where the marginal series follow univariate GARCH processes and the dependence structure between them is specified by a copula function, see e.g. Dias and Embrechts (2004), Rodriguez (2003), Hu (2006), Bartram et al. (2007), Patton (2006), Jondeau and Rockinger (2006), Roch and Alegre (2006), Liu and Luger (2009).

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