

Bayesian Analysis of Simple Random Densities

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Abstract

A tractable nonparametric prior over densities is introduced which is closed under sampling and exhibits proper posterior asymptotics.

Keywords

Bayesian Nonparametrics, Bayesian Density Estimation, Random Densities, Random Partitions, Stochastic Simulations, Smoothing

1. Introduction

The early 1970's witnessed Bayesian inference going nonparametric with the introduction of statistical models with infinite dimensional parameter spaces. The most conspicuous of these models is the Dirichlet process [1], which is a prior on the class of all probability measures over a given sample space that trades great analytical tractability for a reduced support: as shown by Blackwell [2], its realizations are, almost surely, discrete probability measures. The posterior expectation of a Dirichlet process is a probability measure that gives positive mass to each observed value in the sample, making the plain Dirichlet process unsuitable to handle inferential problems such as density estimation. Many extensions and alternatives to the Dirichlet process have been proposed [3].

In this paper we construct a prior distribution over the class of densities with respect to Lebesgue measure. Given a partition in subintervals of a bounded interval of the real line, we define a random density whose realizations have a constant value on each subinterval of the partition. The distribution of the values of the random density on each subinterval is specified by transforming and conditioning a multivariate normal distribution.

Our simple random density is the finite dimensional analogue of the stochastic processes introduced by Thorburn [4] and Lenk [5]. Computations with these stochastic processes involve an intractable normalization constant, and are restricted to values of the random density on a finite number of arbitrarily chosen domain