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MESTRADO PROFISSIONAL EM ECONOMIA

THALES CARLONI GASPAR

INCENTIVE CONTRACTS: STOCKS VERSUS OPTIONS UNDER AMBIGUITY

SÃO PAULO
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Dissertação do Mestrado Profissional em Economia do Insper, apresentado para obtenção do título de Mestre em Economia.

Área de concentração: Microeconomia.

Orientador: Prof. Dr. José Heleno Faro

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RESUMO

O trabalho analisa o custo relativo entre a utilização de ações e opções como base para contratos de incentivo, quando o Principal é neutro ao risco e neutro à ambiguidade e o Agente é avesso ao risco e avesso à ambiguidade. Uma extensão do modelo clássico com função media-variância de preferências é utilizado para lidar com a aversão à ambiguidade. Os resultados indicam que o resultado encontrado no trabalho clássico sobre o tema, onde ações são menos custosas que opções para induzir o nível esperado de esforço por parte do Agente, é válido sob condições gerais. Contudo, existe um cenário possível onde esse resultado é alterado. Ainda, isso só é possível sob um determinado conjunto de restrições e, adicionalmente, tanto o valor da firma quanto o risco do valor da firma são ambíguos.

Palavras-chave: Contrato de Incentivo, Remuneração, Ações e Opções, Ambiguidade.

ABSTRACT

This work analyzes the relative cost of using stocks versus using options on incentive contracts, when the principal is risk and ambiguity neutral and the agent is both risk and ambiguity averse. An extension to the classical mean-variance preference function is used to deal with the ambiguity aversion in the model. It has been found that the result from the classical work, where stocks are less costly then options to induce implementable effort, holds under general assumptions. However, there is a possible scenario where the result is changed. Nevertheless, that is only possible if, among other restrictions, both the firm's mean terminal value and the firm's risk are ambiguous.

Keywords: Incentive Contracts, Compensation, Stocks versus Options, Ambiguity.

EXECUTIVE SUMMARY

Every firm maximizing profits is interested in enhancing their employee's performance. Since profits are affected by employee's productivity and, to achieve the optimal level, firms must be able to create the correct incentives to obtain maximum effort from them when performing their tasks. For that matter, modern organizations have long been developing ways of motivating their employees.

The incentive contracts economic theory highlight that a firm's stock price is often used to measure performance and determine compensation of managers because the incentive devices based on that align the interests of the firm's management to get higher compensation, with the equity-holders desire of maximizing the value of their shares. These incentive devices can be stock-based or option-based. Moreover, classical economic models deal with the situation where a risk averse manager chooses his effort level in order to maximize his expected utility under an option-based incentive contract versus a stock-based one, and concludes that stocks are less costly than options whenever the manager has little or no effect on the firm's operating risk.

Nonetheless, the Bayesian approach of expected utility, which holds that any source of uncertainty can and should be quantified probabilistically, is limited due to it's inability to express the agent's ignorance regarding probability distributions. Economic experiments leaded to the Ellsberg Paradox, revealing the phenomenon of ambiguity aversion, where situations with known instead of unknown probabilities are preferred by people.

The classical economic models are based on expected utility and do not take into consideration the situation when the information over the firm's future performance is ambiguous, and the manager is ambiguity averse. A further study apply the so called Maxmin model to deal with ambiguity, over the classical expected utility model, and state that under some conditions the cost of option compensation can be lower than the cost of stocks, for a specific range of the agent's risk aversion.

The introduction of ambiguity aversion to the classical model provides a broader analysis of the effectiveness of the use of stocks versus options. However, since Maxmin model deals with a worst-case scenario only and the classical model is based on a mean-variance objective function, a different approach appears to be more suited to deal with ambiguity in this situation. The smooth ambuiguity preferences, alongside the approach of the extended mean-variance preference function, has a straightforward fit to the classical model at a first glance. In that framework, the risk aversion term is derived in the mean-variance function when the probabilistic model governing the occurrence of events is uncertain, and an extension to the mean-variance function containing a term to deal with ambiguity is obtained, with an ambiguity aversion parameter.

In this work the extended mean-variance function approach of dealing with ambiguity is applied to the classical model, and the result is compared with both the conclusion based on the agent's expected utility under risk aversion and the conclusion obtained with the introduction of ambiguity using the Maxmin model.

As a result, it has been found that the classical result based on the expected utility holds under a great number of situations, but there is a possible scenario under which option-based contracts are less costly than stock-based ones. An example of that situation is the case where the risk between probability distributions is at least four times greater with stocks than with options, and there is low ambiguity regarding possible probability distributions, combined with a risk neutral agent, who's ambiguity averse. Also, there must be ambiguity regarding both the mean and the risk of the firm's value for the existence of such possibility.

Finally, it has also been found that the cost of using options instead of stocks increases when the agent's risk aversion increases relatively to the ambiguity aversion. Moreover, when there is ambiguity only regarding the firm's risk, which is the case where the Maxmin model is applied, the classical result holds, and stocks are always less costly than options.

Summary

1. Introduction

Every firm maximizing profits is interested in enhancing their employee’s performance. Profits are affected by employee’s productivity and, in order to achieve the optimal level, firms must be able to create the correct incentives to obtain maximum effort from them when performing their tasks. For that matter, modern organizations have long been developing ways of motivating their employees.

Feltham and Wu (2001) highlight that a firm’s stock price is often used to measure performance and determine compensation of managers because the incentive devices based on that align the interests of the firm’s management to get higher compensation, with the equity-holders desire of maximizing the value of their shares. The authors classify the incentive devices as of stock-based or option-based. Moreover, a mean-variance objective function is used to analyze how a risk averse manager chooses his effort level in order to maximize his expected utility under an option-based incentive contract versus a stock-based one, and concludes that stocks are less costly than options whenever the manager has little or no effect on the firm’s operating risk.

Nonetheless, Gilboa and Marinacci (2013) highlight that the Bayesian approach of expected utility, which holds that any source of uncertainty can and should be quantified probabilistically, is limited due to it’s inability to express the agent’s ignorance regarding probability distributions. Ellsberg (1961) experiments leaded to the Ellsberg Paradox and revealed the phenomenon of ambiguity aversion, where situations with known instead of unknown probabilities are preferred by people.

The analysis conducted by Feltham and Wu (2001) is based on expected utility and does not take into consideration the situation when the information over the firm’s future performance is ambiguous, and the manager is ambiguity averse. Liu, Lu and Sun (2017) apply the Maxmin Expected Utility model from Gilboa and Schmeidler (1989), to the agent’s mean-variance preference and find that under some conditions the cost of option compensation can be lower than the cost of stocks, for a specific range of the agent’s risk aversion.

The introduction of ambiguity aversion to the classical model proposed by Feltham and Wu (2001) provides a broader analysis of the effectiveness of the use of stocks versus options. However, since Maxmin model deals with a worst-case scenario only and the classical model is based on a mean-variance objective function, a different approach appears to be more suited to deal with ambiguity in this situation. The smooth ambuiguity preferences defined by Klibanoff, Marinacci and Mukerj (2005), alongside with the approach proposed by Maccheroni, Marinacci and Ruffino (2013), to the mean-variance preference function, has a straightforward fit to the classical model at a first glance. Maccheroni, Marinacci and Ruffino (2013) derive the risk aversion term in the mean-variance function when the probabilistic model governing the occurrence of events is uncertain, and obtain an extension to the mean-variance function containing a term to deal with ambiguity, with an ambiguity aversion parameter.

In this work the Maccheroni, Marinacci and Ruffino (2013) approach of dealing with ambiguity in a mean-variance function is applied to the classical model proposed by Feltham and Wu (2001), and the result is compared with both the conclusion based on the agent's expected utility under risk aversion and the conclusion obtained by Liu, Lu and Sun (2017) with the introduction of ambiguity using the Maxmin model.

A similar methodology has been used by Branger et al. (2017) when analyzing the optimal insurance and saving level for a risk averse agent, using mean-variance preferences with two risk dimensions, the risk within period and the risk between periods. Branger et al. (2017) also use the Maccheroni, Marinacci and Ruffino (2013) approach to mean-variance preferences and conclude that the optimal insurance level is independent of the risk aversion towards between period variance, while the optimal saving level is not. For the optimal insurance level, the result is then the same found by the expected value approach, while the optimal saving level may be affected by the risk aversion towards between periods, depending on the agent's subjective discount factor over time.

In this work we find that the classical result found by Feltham and Wu (2001) holds under a great number of situations, but there is a possible scenario under which option-based contracts are less costly than stock-based ones. An example of that situation is the case where the risk between probability distributions is at least four times greater with stocks than with options, and there is low ambiguity regarding possible probability distributions, combined with a risk neutral agent, who's ambiguity averse. Also, there must be ambiguity regarding both the mean and the risk of the firm's value. Finally, it has also been found that the cost of using options instead of stocks increases when the agent's risk aversion increases relatively to the ambiguity aversion, that when there is ambiguity only regarding the firm's risk, which is the case in Liu, Lu and Sun (2017), the classical result holds, and stocks are always less costly than options.

2. Review of Compensation and Incentive Contracts Literature

Monetary incentives could solve the problem of motivation based on the introductory microeconomic theory, but it has already been shown that there are many other factors involved in the motivation problem. Kamenica (2012) analyses empirical evidence that shows how monetary incentives can backfire in several situations, while nonstandard intervention, such as framing, can be an effective way of influencing behavior. Bénabou and Tirole (2003) discuss the effects of intrinsic and extrinsic motivation, since rewards carry hidden costs and may backfire, especially on the long run, while intrinsic motivation, largely neglected by economists, is often quite rational and plays a central role in many social and economic interactions. Finally, Locke and Latham (2002) provide a well-developed goal-setting theory that emphasizes the important relationship between goals and performance.

When it comes to rewards, monetary incentives cannot be ignored as a very important part of the motivation problem. Despite showing that it can backfire and be counterproductive in some cases, Kamenica (2012) also presents evidence that in many cases it is a powerful tool to improve performance. Also, Gneezy and Rustichini (2000) conducted experiments where it was found that larger amounts of money produced higher performance, except when paying too little, which is worse than not paying at all. Finally, Holmstrom and Milgrom (1991) point that in a standard Principal-Agent model, the compensation system not only rewards productivity, but is also useful in allocating risks, and a conflict only arises when the agent is risk averse. In addition, in the case of multitask, incentive pay works not only to allocate risk and motivate hard work, but also to direct the allocation of the agent's attention to a task among other duties. The first aspect that has to be analyzed is the design of rewards that the worker receives for performing his tasks. There are many possible compensation forms, including pension or health benefits, providing cars for the workers, making monetary payments only, stocks, options, among others.

From the firm's view, Lazear (1986) develops a model to analyze the implications of fixed salaries and piece rate compensations, and concludes that if there are two kinds of firms in the industry, one type paying fixed salaries and another paying piece rate, the low ability workers will always be at the firm that pays fixed salaries. Also, Lazear (1986) points out that piece rate compensation is always more efficient, except if monitoring output costs are too high, and finally, develops a simple model where it is shown that quality is not affected when output is measured on quality characteristics as well as quantity characteristics. Moreover, Paarsch and Shearer (2000) collected data from a tree-planting firm in British Columbia during a period of 6 months and built a model to find the structural effect of incentives of a piece rate compensation model. They found that 22.6% of the total productivity gain was a consequence of the increase in effort due to incentives, but at the same time there was some loss on the average quality, since the number of well planted trees increased only by 14.3%. Furthermore, Lazear (2000) analyses the performance of a large auto glass company over a period of 19 months, during the years of 1994 and 1995, and found out that a change from fixed salaries compensation to piece-rate compensations improved the output per worker by 44%. In the same direction, Shearer (2004) conducted a field experiment in a tree-planting company in British Columbia, changing the compensation method of workers without their knowledge of the experiment in place, and estimated the productivity gain by 20%. After generalizing the experimental results and conducting an out of sample analysis, the conclusion is that the average productivity gain is at least 21.7%.

From que worker's view, Lazear (2000) points out that the change to piece-rate compensation on a large auto glass company rose worker compensation by 7% in the firm, while 92% of workers experienced pay increase, with a quarter receiving increases of at least 28%. Seiler (1984) examined the effect of piece rates on workers earnings, using data from two U.S. manufacturing industries for over 100.000 employees in 500 firms, and found out that incentive workers received an earning premium of 14%. Also, Booth and Frank (1999) build a theoretical model to analyze the relationship between earnings and productivity and test it against data from the British Household Panel Survey, pointing that piece rates increase wages by 9% for men and 6% for women. Nevertheless, contracts might need to cover aspects such as quality, as discussed in Lazear (1986), and Paarsch and Shearer (2000), and recognize that part of the pay increase might be necessary to compensate the greater variation of worker's income, when they are risk averse, as pointed out by Seiler (1984).

Setting goals, on the other hand, is a way of signaling to the employee what is expected from him by the firm. It is also helpful in lowering the worker's uncertainty about his future rewards, since it can be defined as a function of his success achieving the goal. Nevertheless, as mentioned above, rewards are not only monetary and motivation is not only extrinsic, so goal-setting becomes a complex task. Moreover, Locke and Latham (2002) brings together their main findings based on empirical research of what to consider when attempting to use goal-setting to enhance motivation and performance: (i) the highest or most difficult goals produced the highest levels of effort and performance; (ii) specific, difficult goals consistently led to higher performance than urging people to do their best; (iii) Tight deadlines lead to a more rapid work pace than loose deadlines; (iv) goal–performance relationship is strongest when people are committed to their goals; (v) For goals to be effective, people need summary feedback that reveals progress in relation to their goals; (vi) on tasks that are complex for people, learning goals can be superior to performance goals; (vii) proximal goals increase performance when the environment is characterized by uncertainty. Also, Locke and Latham (2007) add two more conclusions based on empirical research: (vii) a positively framed goal produces higher performance than a negatively framed goal; (viii) goals have to be attainable. All these findings bring to light how difficult it is to set goals efficiently, but at the same time provide important tolls that are broadly applicable into modern organizations.

Notwithstanding, all the powerful information presented above does not prevent the firm from incurring into mistakes when setting goals. And even if well designed, the goal may always happen to be too difficult or too easy in the end. The problem of a goal that is too difficult is when it affects commitment to the goal, like when the employee considers it unattainable for example, affecting motivation and leading to lower performance. On the other hand, the goal that's too easy may be an issue since it could be accepted by an employee willing to minimize his effort level instead of maximizing achievements and rewards. On that matter, besides conclusion (i) above, presented by Locke and Latham (2002), also Latham and Yukl (1975) found evidence that inside organizations, both a no-goal and an easy-goal situation lead worse performance than a high-goal situation.

The firm will also have to take other aspects into consideration when facing a two-period or even a multi-period situation. First, all goals must be set for a specific time period. That is the time the employee will have to work on his tasks and over which the firm will evaluate his achievements. Second, that time period cannot be too long, in accordance with conclusions (iii) and (vii) above. Then, it is likely that an employee will work over several different periods, with goals set for each of them during the time working for a firm, and as a consequence to that, one period's goal and performance may influence his expectancy and performance for the next period.

A firm willing to set goals may look forward, and take into consideration what will the economic environment be during the period to which goals will be assigned, since it will impact the achievements of employees. Nonetheless, there is uncertainty about that economic environment and also on other aspects involved on employee's performance, beyond personal effort. Another problem that may arise from this approach is that the firm might not be completely aware of what the actual potential of a specific business unit is, especially when it is a new project or if the market has changed significantly with respect to that activity. In that case, firm may set next period's goals based on the available information about past periods agent's performance. When that happens, there is a chance the worker may intentionally depress the output on this period in order to avoid a more difficult goal on the next period, and the potential negative effect of that on compensation. On that matter, Lazear (1986) conduces the analysis of a two-period model on how the worker maximizes his expected utility when considering both the aggregate compensation and the cost-to-effort function. The main findings are that the intertemporal strategic behavior does not affect piece rate compensation efficiency, and that paying a higher piece rate on the first period creates the right incentives to avoid workers from depressing first period's output. However, as mentioned above, the worker may face several consecutive periods while the time working at the same firm, and the two-period model does not address this issue, especially when the number of periods that the worker will stay at the firm is unknown. Gibbons and Murphy (1992) propose that when there are career concerns in place, compensation contracts should be stronger for workers closer to retirement, due to their weaker concern about their career. An interesting result from their work is that this prediction holds when analyzing data from the relation between CEO's compensation and stock market performance during the 70's and 80's.

The conclusion from Gibbons and Murphy (1992) does not only address career concerns, but also brings to light the positive relation between stock market performance and worker's compensation. A possible way to bring shareholder's and worker's interests altogether is to set stock price based contracts. In that direction, Holmstrom and Milgrom (1991) show that when the agent owns the asset returns, the incentive contract provides more intensive incentives to engage in production. Also, Holmstrom and Milgrom (1994) point out that asset ownership is a powerful instrument to deal with the existence of performance high monitoring costs. Due to the strong alignment between shareholder's and workers interests with stock price based contracts, those will be the focus of this work, more specifically over its shape, since it can be either stock or option based.

Since it relies on the shareholders to define whether to grant stocks or options to the worker, the choice will always be the one that maximizes the shareholders utility. That is a Principal-Agent situation where the shareholders represent the principal and the worker is the agent. The principal wants to maximize the firm value, which is a function of the worker's level of effort, and minimize the cost of compensating the agent for the that effort level. Then, the shape of the incentive contract will be the one minimizing the cost to the principal on a situation where maximum implementable effort by the agent is induced. Hall and Murphy (2000) conduct an analysis on the optimal price of stock options granted to managers and conclude that pay-to-performance incentives are maximized when options are issued at-the-money, which means when the exercise price is equal to the stock's current price, or near that point. Finally, the solution to this problem lies on finding if a contract based on stocks or options is less costly to the principal for the same level of incentive to the agent.

3. Related Works

3.1. The Benchmark Model with Expected Utility Agents

The basic model analyzed by Feltham and Wu (2001) consists on a principal-agent model, where the principal acts of behalf of the firm's owners, who are risk neutral, and the agent is the manager, hired to operate the firm for a single period. The manager is offered a contract at *t = 0*, and the firm's value at *t = 1* is represented by $\tilde{x}$, which is normally distributed with average $m_x = m_x(a)$ and standard deviation $\sigma_x = \sigma_x(a)$, both functions of the manager's effort level *a ϵ A*, unobservable to the principal.

The owners hold and option on the firm's value, with exercise price equal to zero, so they receive max $\{x\,,0\}$. However, option can be issued with exercise price $k \geq 0$, where the option holder receives max $\{x - k\,,0\}$, and the stock is the particular case where $k = 0$.

In that context, the mean and variance of an option on the firm's value at *t = 1*, with exercise price k are given by

$$m_k(a) = \sigma_x(a)[\phi - (1 - \Phi)\xi], \quad (1)$$

$$\sigma_k^2(a) = \sigma_x^2(a)[1 - \Phi - \phi^2) - \phi(2\Phi - 1)\xi + \Phi(1 - \Phi)\xi^2], \quad (2)$$

Where,

$$\xi = \frac{k - m_x(a)}{\sigma_x(a)}$$

$$\phi = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\xi^2\right]$$

$$\Phi = \int_{-\infty}^{\xi} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}t^2\right] dt$$

which are, respectively, the number of standard deviations between k and $m_x(a)$, the standard normal probability density function at ξ, and the standard normal cumulative probability function at ξ.

Feltham and Wu (2001) show that both the expected value and variance of an option increase with m_x and σ_x, and decrease with k.

Moreover, the manager will choose his level of effort $a \epsilon A$ that maximizes his expected utility, with respect to $\tilde{w} - g(a)$, where $\tilde{w}$ is the manager's terminal wealth and $g(a)$ the cost of the effort level a. The agent is risk averse and has his preferences represented by a mean variance "certainty-equivalent" objective function,

$$\mathrm{CE}(a, c) = \mathrm{E}[\tilde{w}|a, c] - \frac{1}{2}\lambda \mathrm{Var}[\tilde{w}|a, c] - g(a), \tag{3}$$

where *c* represents the compensation contract, $g(a) = \frac{1}{2}a^2$, and λ is the risk aversion parameter.

The principal is risk neutral, and maximizes the expected firm value, after the agent's payment,

$$\underset{a,c}{\text{maximize}}\ \mathrm{E}[\tilde{x} - \tilde{w}|a, c] \tag{4}$$

$$\text{subject to: } \mathrm{CE}(a, c) \geq \mathrm{CE}^0$$

$$a\ \epsilon \arg\max_{\hat{a}} \mathrm{CE}(\hat{a}, c).$$

Feltham and Wu (2001) conduct a detailed analysis on the costs of induce effort, and conclude that the following proposition holds:

Proposition 1. *Assume the manager is strictly risk averse, his effort has a linear impact on the mean of $\tilde{x}$ but not on the variance of $\tilde{x}$, his personal cost of effort is quadratic, and his compensation consists of a fixed wage plus a grant of options with exercise price $k \geq 0$. Under these conditions, the cost of inducting the manager to select an implementable action a increases with the exercise price k and, hence, is minimized by issuing stock (for which $k = 0$).*

3.2. Ambiguity and Expected Utility

Gilboa and Marinacci (2013) highlight that during the last century the economic theory has been dominated by the Bayesian paradigm, which holds that all uncertainty can and should be quantified probabilistically. That means a decision maker, in the absence of objective probabilities, will always have subjective probabilities to guide decisions. However, such axiom fails to reflect situations where the agent is ignorant about probabilities, and once forced to choose a probability, the choice may simply be arbitrary. Finally, the classic expected utility approach is based on the independence axiom. This axiom states that the agent's preference between two acts is not affected by mixing those acts with a third one. Notwithstanding, it fails to deal with situations where mixing acts lowers uncertainty, like hedging strategies as an example.

Further on, Ellsberg (1961) conducted experiments involving urns and balls, and found out that there was greater preference for the situation where the probability distribution was known by the agents, when compared to unknown ones, even when the expected value was identical. That became known as the Ellsberg Paradox, and brought to light the phenomenon of ambiguity aversion.

Gilboa and Schmeidler (1989) developed an axiomatic model to deal with ambiguity, known as Maxmin, which consists on maximizing the minimum expected value by the decision maker. When the agent does not know the probability distribution for a certain event, dealing with the worst-case scenario is a way of finding protection for uncertainty, and is a way of dealing with ambiguity aversion.

However, ambiguity aversion might not be so extreme that the agent seeks to maximize its minimum expected utility, ignoring all other possible scenarios. Klibanoff, Marinacci and Mukerji (2005) introduced a different relation of preferences regarding ambiguity, known as smooth ambiguity. Under this framework, preferences are a response to risk aversion and ambiguity aversion in two different dimensions, continuously, and defined by individual parameters. The maxmin is then a particular case of the smooth ambiguity when ambiguity aversion is infinite. All that development on how to deal with ambiguity provides powerful tools to deal with situation where the classical expected utility paradigm fails to consider all relevant aspects, as it will be better addressed below.

3.3. Maxmin Ambiguity Aversion Model

Liu, Lu and Sun (2017) took the benchmark model of stocks and options as presented by Feltham and Wu (2001) as a starting point, representing the risk averse agent's expected utility by the mean-variance preference:

$$E[u] = E[w] - \frac{1}{2}\lambda \mathrm{Var}[w] - g(a), \tag{5}$$

Where, w is the agent's compensation, $g(a) = \frac{1}{2}a^2$ is the cost for the effort level a, and λ is the risk aversion parameter.

And the principal objective represented by:

$$\max_{w} E[x - w] \ , \tag{6}$$

subject to $E[u(a)] \geq 0$, and

$a \,\epsilon \arg\max_{\hat{a}} E[u(\hat{a})]$.

which is very similar to (4), with $\mathrm{CE}^0 = 0$. The firm value was set as $x = a + l + \varepsilon$, where ε is the noise term, normally distributed with mean zero and variance σ^2, and l is a constant term. Then, the target effort is fixed, and the above Proposition 1 is presented as:

Denote C_s as the cost of stock compensation to implement the target effort a, and C_k as the cost of option compensation to implement a. For any $k \geq 0, C_s < C_k$.

From this point, the ambiguity aversion is included in the model, and the agent minimizes his utility for any possible distributions, considering the worst-case scenario, and then chooses his effort level that maximizes his utility, which has been minimized for the possible distributions. The agent's objective is denoted by:

$$\max_{w} \min_{\sigma \epsilon [\sigma_1, \sigma_2]} E[u(\hat{a})] \tag{7}$$

Liu, Lu and Sun (2017) assume that the there is no asymmetry between the principal and the agent regarding the risk range, so the principal's perception of risk σ_p, can be denoted by $\sigma_p \,\epsilon\, [\sigma_1, \sigma_2]$, but the mean is known, which is a very particular ambiguity situation. Also, since the principal is able to diversify her risk in the market, she is less averse to ambiguity on risk. There are no assumptions on the principal's ambiguity aversion, but the analysis consists on the fact that the agent is ambiguity averse. Furthermore, the authors state:

Lemma 2. *Let σ_a and σ_p be the agent's and the principal's perception of risk, respectively.*

1) If the principal uses stock compensation to induce effort, then the agent's perception of risk is $\sigma_p = \sigma_2$; the cost of stock compensation is $C_s = \frac{1}{2}\lambda a^2 \sigma_2^2$.

2) If the principal uses option compensation to induce effort, let $\beta_k(\sigma_1)$ be the number of option required to induce the target effort a if the agent's perception of risk is σ_1, i.e., $a \,\epsilon \arg\max_{\hat{a}} E[u(\hat{a})|\sigma_a = \sigma_1 \; and \; \beta_k = \beta_k(\sigma_1)]$. If the agent is granted $\beta_k(\sigma_1)$ option with an exercise price k, and

$$\lambda < \min_{\sigma\epsilon[\sigma_1,\sigma_2]} \frac{2}{\beta_k(\sigma_1)} \frac{m_k(\sigma)-m_k(\sigma_1)}{\sigma_k^2(\sigma)-\sigma_k^2(\sigma_1)}, \tag{8}$$

then the agent will exert the effort a, his perception of risk is σ_1 and the cost of option compensation is $C_s = \frac{1}{2}\lambda\beta_k^2(\sigma_1)\sigma_k^2(\sigma_1) + \beta_k(\sigma_1)[m_k(\sigma) - m_k(\sigma_1)]$.

The main implications of Lemma 2 are that the agent always perceives a higher risk when he owns stocks, but the increase in variance in this case does not increase the mean value of the stocks, while in the case he owns options, the increase in variance increase both the risk and the mean value of options. The upside of increasing the mean value will be greater than the downside from higher risk if the agent is not too risk averse.

Liu, Lu and Sun (2017) argue that since stocks led the agent to perceive a higher risk, the principal will have to pay a higher risk premium to offset that. Notwithstanding, since options may induce the lower risk perception, it not only reduces the risk premium, but it also increases the gap between the principal's and the agent's valuation of options. For certain levels of risk aversion by the agent, the cost of options compensation will decrease. Based on that, the authors state:

Proposition 2. *Suppose σ_1 is small enough such that $a^2\sigma_2^2 > \beta_k^2(\sigma_1)\sigma_k^2(\sigma_1)$. If the agent's risk aversion λ satisfies*

$$\frac{2\beta_k(\sigma_1)[m_k(\sigma_p)-m_k(\sigma_1)]}{a^2\sigma_2^2-\beta_k^2(\sigma_1)\sigma_k^2(\sigma_1)} < \lambda < \min_{\sigma\epsilon[\sigma_1,\sigma_2]} \frac{2}{\beta_k(\sigma_1)}\frac{m_k(\sigma)-m_k(\sigma_1)}{\sigma_k^2(\sigma)-\sigma_k^2(\sigma_1)}, \tag{9}$$

Then it is less costly to use options (whit an exercise price k) than stocks to induce effort.

The proposition above presents a possible scenario under which options can be less costly for the principal. However, there are many assumptions regarding the agent's risk aversion to be met, and even though all conditions fit the model, it is still not clear that the Maxmin model is the best suited for a mean-variance expected utility preference, since it puts a lot of weight on the worst-case scenario.

3.4. Smooth Preferences

Klibanoff, Marinacci and Mukerji (2005) introduced a class of preference relation that characterizes the smooth model of ambiguity (KMM, for short). Preferences are characterized as a functional of the double expectational form:

$$V(f) = \int_\Delta \phi\left(\int_S u(f)\, d\pi\right) d\mu \tag{10}$$

where *f* is a real-valued function defined on a state space S, u is a von Neumann–Morgenstern utility function, π is a probability measure on S, μ is the DM's subjective prior over Δ, the set of possible probabilities π over S. While u characterizes attitude toward pure risk, ambiguity attitude is captured by ϕ. In particular, a concave ϕ characterizes ambiguity aversion.

The Maxmin model can be seen as a particular case of the KMM model where ambiguity aversion is infinite.

3.5. Mean-Variance Ambiguity Approach

Maccheroni, Marinacci and Ruffino (2013) highlight that the Arrow-Pratt approximation of certainty equivalent, for a von Neumann-Morgenstern utility maximizer, with utility *u* and wealth *w*, who considers an investment *h*, is

$$c(w+h,P) \approx w + E_p(h) - \frac{1}{2}\lambda_u(w)\sigma_P^2(h), \tag{11}$$

where *P* is the probabilistic model describing the stochastic nature involved in the problem. That approximation makes the risk of *h* proportional to it's variance, and provides the foundation to the mean-variance preference model:

$$U(f) = E_p(f) - \frac{1}{2}\lambda\sigma_P^2(f), \tag{12}$$

Where $w + h = f$ and $\lambda_u(w) = \lambda$.

To build an extension to the classic Arrow-Pratt approximation, they start by the certain equivalent when only risk is present, and u represents the attitude toward risk.

$$c(w+h,P) = u^{-1}\left(E_p\big(u(w+h)\big)\right) \tag{13}$$

Nevertheless, if there is uncertainty about the true probabilistic model P, the agent may also consider alternative probabilistic models Q, and while u represents his attitude toward risk of P, v represents the attitude toward model uncertainty, the certain equivalent is:

$$C(w+h,P) = v^{-1}\left(E_u\big(c(w+h)\big)\right) = v^{-1}\left(E_\mu\left(v\left(u^{-1}\left(E\big(u(w+h)\big)\right)\right)\right)\right) \tag{14}$$

Where $c(w+h)$ associates $c(w+h,Q)$ to each model Q, resulting in the smooth ambiguity certainty equivalent of Klibanoff, Marinacci, and Mukerji (2005) abbreviated as KMM.

When u differs from v it is not possible to reduce model uncertainty to risk. Maccheroni, Marinacci and Ruffino (2013) derive the approximation of (10) under ambiguity and show that:

$$C(w+h) \approx w + E_{\bar{Q}}(h) - \frac{1}{2}\lambda_u(w)\sigma_{\bar{Q}}^2(h) - \frac{1}{2}(\lambda_v(w) - \lambda_u(w))\sigma_u^2(E(h)) \quad (15)$$

Where, $\bar{Q} = \int Q d\mu(Q)$ is the reduced probability induced by u, and $E(h): \Delta \to \mathbb{R}$ is the random variable $Q \mapsto E_Q(h)$ that associates $E_Q(h)$ to each possible Q. The variance $\sigma_u^2(E(h)$ together with the difference $\lambda_v(w) - \lambda_u(w)$ determine the ambiguity premium.

Setting $w + h = f$, $\lambda_u(w) = \lambda, \lambda_v(w) - \lambda_u(w) = \theta$, and $\bar{Q} = P$:

$$U(f) = E_P(f) - \frac{1}{2}\lambda\sigma_P^2(f) - \frac{1}{2}\theta\sigma_\mu^2(E(f)) \quad (16)$$

Where λ and θ are the parameters for risk and ambiguity aversion, respectively, in this extension of the mean-variance model.

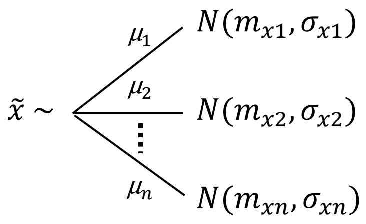
This is the framework over which our analysis of the Stock versus Options mean-variance benchmark model will be conducted to analyze how the insertion of ambiguity aversion affects the results of the classic model.

4. Model

4.1. Benchmark Model with Mean-Variance Function Ambiguity

The extension of the classic mean-variance function presented by Maccheroni, Marinacci and Ruffino (2013) has a straightforward intuition. While the classic mean-variance certainty equivalent function is the expected wealth minus a risk aversion parameter proportional to the wealth's variance, the extended approach adds an ambiguity parameter proportional to the probabilistic model's uncertainty. That means in the presence of ambiguity, the ambiguity aversion reduces the certainty equivalent value in the same way risk aversion does when risk is involved.

In the model $\tilde{x}$ represents the gross terminal value of the firm at *t = 1* and ambiguity is described by a finite family of normal distributions with probability μ_j:



$\tilde{x}$ is a normally distributed random variable with mean $m_{xj} = m_{xj}(a)$ and standard variation $\sigma_{xj} = \sigma_{xj}(a)$, where $a \, \epsilon \, A$.

In that context, the agent's mean-variance certainty equivalent function for a random variable $\tilde{w}$ and action a is given by:

$$\mathrm{CE}(a,c) = \mathrm{E}_P[\tilde{w}|a,c] - \frac{1}{2}\lambda \mathrm{Var}_P[\tilde{w}|a,c] - \frac{1}{2}\theta \mathrm{Var}_\mu(\mathrm{E}_Q[\tilde{w}|a,c]) - g(a), \tag{17}$$

Where $P = \mathrm{E}_\mu[Q]$, $\tilde{w}$ is the agent's compensation, $g(a) = \frac{1}{2}a^2$ is the cost for the effort level a, λ and θ are respectively the risk aversion and ambiguity aversion parameters.

And the principal objective for a risk neutral and ambiguity neutral principal is:

$$\underset{a,c}{\text{maximize}} \; \mathrm{E}[\tilde{x} - \tilde{w}|a,c] \tag{18}$$

subject to: $\mathrm{CE}(a,c) \geq \mathrm{CE}^0$

$a \epsilon \arg\max_{\hat{a}} \mathrm{CE}(\hat{a}, c)$.

A contract of the form $\mathrm{C}_k(\alpha_k, \beta_k)$ is considered and the agent's compensation has the form $w = \alpha_k + \beta_k \max\{\tilde{x} - k, 0\}$, where α_k is a fixed wage and β_k is the number of options the agent receives, and k the is option's exercise price.

Hence,

Proposition 1

$$\mathrm{CE}(a, c) = \alpha_k + \beta_k \bar{m}_k(a) - \tfrac{1}{2}\lambda\beta_k^2\bar{\sigma}_k^2(a) - \tfrac{1}{2}\theta\sigma_\mu^2(\alpha_k + \beta_k m_k(a)) - g(a), \quad (19)$$

Is the agent's certainty equivalent function, where the mean and variance with respect to the average distribution are:

$$\bar{m}_k(a) = \bar{\sigma}_x(a)[\phi - (1 - \Phi)\xi], \quad (20)$$

$$\bar{\sigma}_k^2(a) = \bar{\sigma}_x^2(a)[1 - \Phi - \phi^2) - \phi(2\Phi - 1)\xi + \Phi(1 - \Phi)\xi^2], \quad (21)$$

and,

$$\xi = \frac{k - \bar{m}_x(a)}{\bar{\sigma}_x(a)}$$

$$\phi = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\xi^2\right]$$

$$\Phi = \int_{-\infty}^{\xi} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}t^2\right] dt$$

Since the fixed wage and the number of options does not influence the model variance, the mean-variance preference can be written as:

$$\mathrm{CE}(a, c) = \alpha_k + \beta_k \bar{m}_k(a) - \tfrac{\beta_k^2}{2}\left(\lambda\bar{\sigma}_k^2(a) + \theta\sigma_\mu^2\left(m_k(a)\right)\right) - g(a) \quad (22)$$

Assuming the agent's external opportunity $\mathrm{CE}^0 = 0$, from equation (22) we have that the fixed wage α_k is:

$$\alpha_k = \frac{\beta_k^2}{2}\left(\lambda\bar{\sigma}_k^2(a) + \theta\sigma_\mu^2\big(m_k(a)\big)\right) + g(a) - \beta_k\bar{m}_k(a), \tag{23}$$

we note that:

$$\sigma_\mu^2\big(m_k(a)\big) = \sum_{j=1}^{n}\mu_j\left(m_{kj}(a) - \bar{m}_k(a)\right)^2 \tag{24}$$

The fixed wage compensates the agent for his external opportunity, plus a risk premium, an ambiguity premium, and the cost of effort, minus the value of the options received.

Remark: If the agent is ambiguity neutral, then the result based on the expected utility model holds with $\tilde{x} \sim N(\bar{m}_x, \bar{\sigma}_x)$, where $\bar{m}_x = \sum_{j=1}^{n}\mu_j m_{xj}$ and $\bar{\sigma}_x^2 = \sum_{j=1}^{n}{\mu_j}^2{\sigma_{xj}}^2$.

Moreover, the principal wants to maximize the firm's terminal value net of the agent's compensation, and subject to the agent's participation restriction.

Proposition 2

The principal problem is then:

$$\underset{a,\beta_k,k}{\text{maximize}}\ \bar{m}_k(a) - \left[\frac{\beta_k^2}{2}\left(\lambda\bar{\sigma}_k^2(a) + \theta\sigma_\mu^2\big(m_k(a)\big)\right) + g(a)\right] \tag{25}$$

subject to: $$a \,\epsilon\, \arg\max_{\hat{a}}\ \beta_k\bar{m}_k(\hat{a}) - \frac{\beta_k^2}{2}\left(\lambda\bar{\sigma}_k^2(\hat{a}) + \theta\sigma_\mu^2\big(m_k(\hat{a})\big)\right) - g(\hat{a}) \tag{26}$$

And the first-order condition of (23) with respect to a is:

$$\beta_k\frac{d}{da}\big(\bar{m}_k(a)\big) - \frac{\beta_k^2}{2}\left(\lambda\frac{d}{da}\left(\bar{\sigma}_k^2(a)\right) + \theta\frac{d}{da}\left(\sigma_\mu^2\big(m_k(a)\big)\right)\right) - g'(a) = 0 \tag{27}$$

4.2. Optimal Number of Options to Induce Effort Under Ambiguity

To define whether it is costlier to the principal to use an option-based contract instead of a stock-based contract, it is necessary to find the optimal number of options needed to be granted to the agent, in order to induce implementable effort. The first step is to define when a is implementable, and then find $\beta_k(a)$.

Proposition 3

Assuming that the agent's action is single dimensional, denoted by $a \epsilon [0, \infty)$, and $\forall\, j \in \{1,2, \dots n\}, m_{xj}(a) = m_j + a$, then, $\bar{m}_x(a) = \sum_{j=1}^{n} \mu_j m_{xj}\,(a) = \sum_{j=1}^{n} \mu_j m_j + a = \bar{m} + a$.

and,

$$\frac{d}{da}\big(\bar{m}_k(a)\big) = 1 - \Phi$$

$$g'(a) = a$$

a is implementable if and only if,

$$g'(a) = a \leq G(\xi) = \frac{\frac{d}{da}\big(\bar{m}_k(a)\big)^2}{2\left(\lambda \frac{d}{da}\left(\bar{\sigma}_k^2(a)\right) + \theta \frac{d}{da}\left(\sigma_\mu^2(m_k(a))\right)\right)} \tag{28}$$

In this context,

$$\beta_k(a) = 2a\left[1 - \Phi + \sqrt{(1-\Phi)^2 - 2a\left(\lambda \frac{d}{da}\left(\bar{\sigma}_k^2(a)\right) + \theta \frac{d}{da}\left(\sigma_\mu^2\big(m_k(a)\big)\right)\right)}\,\right]^{-1} \tag{29}$$

Since both the expected value and variance of an option decrease with k, while $\beta_k(a)$ in equation (29) increases with k, λ and θ, $G(\xi)$ from equation (28) decreases with the same variables.

When granted options, the manager faces higher mean and variance of his wealth. Analyzing $G(\xi)$, it can be seen that for a risk averse and ambiguity averse manager, the effort level will not be increased by a larger number of options, since the marginal benefit on the mean of his wealth will be offset by the marginal increase in the variance. With ambiguity, the effect is even stronger, because $G(\xi)$ also decreases with the parameter θ, and the variance between possible probability distributions and the average distribution.

4.3. Comparison of Stocks and Options Based Contracts Under Ambiguity

A stock based contract is a particular case of an option based contract where $k = 0$. Since the agent's level of effort does not influence risk, it does not influence ambiguity either, then $\bar{m}_0(a) \approx \bar{m} + a$, $\frac{d}{da}\big(\bar{m}_0(a)\big) \approx 1$, $\bar{\sigma}_0^2(a) \approx \bar{\sigma}^2$, $\frac{d}{da}\big(\bar{\sigma}_0^2(a)\big) \approx 0$, $\sigma_\mu^2\big(m_0(a)\big) \approx \sum_{j=1}^{n} \mu_j^2\, \sigma_{\mu j}^2$, and $\frac{d}{da}\Big(\sigma_\mu^2\big(m_0(a)\big)\Big) \approx 0$.

In that context, from equation (29) we find that the number of stocks to induce implementable effort is:

$$\beta_0(a) \approx a. \tag{30}$$

That means the required number of stocks to be granted to the agent is independent of the variance, the agent's risk and ambiguity aversion. The action taken by the worker will be so that the marginal benefit from receiving the stocks equals the marginal cost of effort. More effort can though be induced by increasing the marginal benefit, and there is no limit on that.

Assuming options are issued at-the-money, $\hat{k} = \bar{m}_0(a) \approx \bar{m}_x(a)$, and as a consequence $\xi \approx 0$, $\phi \approx \frac{1}{\sqrt{2\pi}}$, and $\Phi \approx \frac{1}{2}$. Under these circumstances the optimal number of options is:

$$\beta_k(a) \approx 2a\left[1 - \frac{1}{2} + \sqrt{\left(\frac{1}{2}\right)^2 - 2a\left(\lambda \frac{d}{da}\left(\bar{\sigma}_k^2(a)\right) + \theta \frac{d}{da}\left(\sigma_\mu^2\big(m_k(a)\big)\right)\right)}\right]^{-1} \tag{31}$$

Note that when the agent is risk neutral and ambiguity neutral,

$$\beta_k(a) \approx 2a\left[1 - \frac{1}{2} + \sqrt{\left(\frac{1}{2}\right)^2}\right]^{-1} \approx 2a \tag{32}$$

Equation (32) shows that when the agent is risk neutral and ambiguity neutral the number of options granted is twice the number of stocks. Since $\beta_k(a)$ increases with λ and θ, for a risk averse or/and ambiguity averse agent, the number of options would be even greater than the number of stocks.

The economic intuition behind this result is straightforward. When holding options, the agent's payoff is zero whenever the stock price is below the exercise price, and positive when above. With stocks, the payoff is always positive, unless the firm falls into bankruptcy and the stock price is zero. Then, the number of options has to be greater to achieve the same expected payoff when compared to stocks, to compensate for all the scenarios where the payoff for holding options will be equal to zero.

Moreover, the manager's compensation risk has to be analyzed on both situations, since the optimal number of options and stocks need to achieve the same expected payoff, increase in the same direction that risk and ambiguity do. From equation (31) there are two risk components, $f_k^2(a) = \beta_k^2(a)\bar{\sigma}_k^2(a)$, related to the risk of the mean probability distribution, and $y_k^2(a) = \beta_k^2(a)\sigma_\mu^2(m_k(a))$, related to the risk between probability distributions. With options we have,

$$f_k^2(a) \approx 4a^2\bar{\sigma}^2\left[\frac{1}{2} + \left(\frac{1}{\sqrt{2\pi}}\right)^2\right] = a^2\bar{\sigma}^2\left[4\left(\frac{1}{2} - \frac{1}{2\pi}\right)\right] > a^2\bar{\sigma}^2 \approx f_0^2(a) \quad (33)$$

Resulting the mean probability distribution risk is greater with options. And,

$$y_k^2(a) \approx 4a^2\sigma_\mu^2\big(m_k(a)\big) = 4a^2\left(\sum_{j=1}^n \mu_j\left(m_{\hat{k}j}(a) - \bar{m}_{\hat{k}}(a)\right)^2\right) \quad (34)$$

While,

$$y_0^2(a) \approx a^2\left(\sum_{j=1}^n \mu_j^2\,\sigma_{\mu j}^2\right) = a^2\left(\sum_{j=1}^n \mu_j^2\,\frac{\left(m_{0j}(a) - \bar{m}_0(a)\right)^2}{n-1}\right) \quad (35)$$

Meaning the manager's compensation risk between probability distributions is greater with options when $y_k^2(a) > y_0^2(a)$.

To analyze this inequality, we start by supposing that it holds, which implies that,

$$4a^2\left(\sum_{j=1}^n \mu_j\left(m_{\hat{k}j}(a) - \bar{m}_{\hat{k}}(a)\right)^2\right) > a^2\left(\sum_{j=1}^n \mu_j^2\,\frac{\left(m_{0j}(a) - \bar{m}_0(a)\right)^2}{n-1}\right) \quad (36)$$

Since $\mu_j > \mu_j^2$, equation (36) is true whenever,

Theorem 1

Stock-based contracts are less costly than option-based contracts when

$$\sum_{j=1}^{n}\left(m_{\hat{k}j}(a)-\bar{m}_{\hat{k}}(a)\right)^{2} > \frac{\sum_{j=1}^{n}\left(m_{0j}(a)-\bar{m}_{0}(a)\right)^{2}}{4(n-1)} \tag{37}$$

If there is ambiguity, $n \geq 2$, meaning equation (37) always holds if the variance between probability distributions, with options, is greater than a quarter of the variance with stocks. Also, note that when there is too much ambiguity, i.e. $n \rightarrow \infty$, the right hand of equation (37) tends to 0. Furthermore, the number 4 in the denominator of the right hand is a result from equation (32), which is the extreme case where the agent is both risk and ambiguity neutral. and that number increases when the agent is risk averse or/and ambiguity averse. Since $\beta_0(a)$ is independant of λ and θ, and $\beta_k(a)$ is increasing in k, λ and θ, for extreme high values of λ and θ, the right hand of equation (37) will also tend to zero, and the inequality will hold. Then the manager's compensation risk is greater with options, when compared to stocks., and the cost of using at-the-money options to motive a manager who is risk and ambiguity averse is greater than in the situation where stocks are used.

Notwithstanding, even if the inequality in equation (37) does not hold, the result remains unchanged whenever $\lambda f_k^2(a) > \theta y_k^2(a)$, for all values of k. That means that even in the case where the compensation risk between probability distributions is greater with stocks, $y_k^2(a) < y_0^2(a)$, the result is not affected if the compensation risk of the mean probability distribution, $f_k^2(a)$, multiplied by the risk aversion parameter, λ, is greater than the compensation risk between probability distributions, $y_0^2(a)$, multiplied by the ambiguity aversion parameter, θ, which can also be expressed by,

$$\lambda\left[\bar{\sigma}^2\left(\frac{1}{2}-\frac{1}{2\pi}\right)\right] > \theta\left[\sum_{j=1}^{n}\mu_j\left(m_{kj}(a)-\bar{m}_k(a)\right)^2\right] \tag{38}$$

This equation brings to light two important conclusions. First, the cost of stocks decreases when the risk aversion increases relatively to the ambiguity aversion, which is expressed by λ relatively to θ in the model. Second, stocks are less costly whenever there is ambiguity only regarding the variance of the true probability distribution. If there is no ambiguity regarding the mean, then $m_{kj}(a) = \bar{m}_k(a)$, and,

$$\left[\sum_{j=1}^{n} \mu_j \left(m_{kj}(a) - \overline{m}_k(a)\right)^2\right] = 0 \quad (39)$$

In this situation, the inequality in equation (38) always holds, and stock-based contracts are always less costly than option-based contracts.

Finally, there is a possible scenario where stock-based contracts are costlier than option-based contracts. That is the case when both equations (37) and (38) do not hold. A possible example of this case would be a scenario where the variance between probability distributions with stocks is higher than with options, and there is low ambiguity, leading to lower values of n in equation (37), combined to the situation where the agent is not risk averse, but is ambiguity averse, meaning the left hand of equation (38) is zero.

5. Conclusion

The cost to induce implementable effort by the agent has been compared in the situation where the principal grants stocks versus the situation where at-the-money options are granted, when the agent is not only risk averse but also ambiguity averse. The analysis is based on the classical approach to the subject where mean-variance preferences are used, with an extension to deal with the ambiguity aversion in the mean-variance function framework.

It has been found that under general assumptions the result found by the expected utility model holds. However, there may be a very particular case where granting stocks is costlier than options. Notwithstanding, among other restrictions, there must be ambiguity regarding the mean and variance of the firm’s terminal value.

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APPENDIX

Proof of Proposition 1:

The mean $\overline{m}_k(a)$ and the variance $\bar{\sigma}_k^2(a)$ are the result of a truncated normal distribution, as shown by theorem 22.3 in Greene (2002).

The agent's certainty equivalent is:

$$\mathrm{CE}(a, c_k) = E_P[\alpha_k + \beta_k max\{x-k,0\}] - \frac{1}{2}\lambda Var_P[\alpha_k + \beta_k max\{x-k,0\}] - \frac{1}{2}\theta Var_\mu\Big[E_Q[\alpha_k + \beta_k max\{x-k,0\}]\Big] - g(a)$$

$$= \alpha_k + \beta_k E_P[max\{x-k,0\}] - \frac{1}{2}\lambda\beta_k^2 Var_P[max\{x-k,0\}] - \frac{1}{2}\theta\beta_k^2 Var_\mu[m_{k1}(a), m_{k2}(a), \ldots, m_{kn}(a)\ ] - g(a)$$

$$= \alpha_k + \beta_k\overline{m}_k(a)\ - \frac{1}{2}\lambda\beta_k^2\bar{\sigma}_k^2(a) - \frac{1}{2}\theta\beta_k^2\sigma_\mu^2\big(m_k(a)\big) - g(a)$$

$$= \alpha_k + \beta_k\overline{m}_k(a) - \frac{\beta_k^2}{2}\Big(\lambda\bar{\sigma}_k^2(a) + \theta\sigma_\mu^2\big(m_k(a)\big)\Big) - g(a)$$

Proof of Proposition 3:

Note that equation (27) is a quadratic function on β_k, where:

$$a = -\frac{1}{2}\left(\lambda\frac{d}{da}\left(\bar{\sigma}_k^2(a)\right) + \theta\frac{d}{da}\left(\sigma_\mu^2\big(m_k(a)\big)\right)\right)$$

$$b = \frac{d}{da}\big(\overline{m}_k(a)\big)$$

$$c = g'(a)$$

Hence,

$$b^2 - 4ac = \frac{d}{da}\big(\overline{m}_k(a)\big)^2 - 4\left(-\frac{1}{2}\right)\left(\lambda\frac{d}{da}\left(\bar{\sigma}_k^2(a)\right) + \theta\frac{d}{da}\left(\sigma_\mu^2\big(m_k(a)\big)\right)\right)\ g'(a)$$

$$b^2 - 4ac \geq 0 \iff \frac{d}{da}\left(\bar{m}_k(a)\right)^2 - 2\left(\lambda \frac{d}{da}\left(\bar{\sigma}_k^2(a)\right) + \theta \frac{d}{da}\left(\sigma_\mu^2\left(m_k(a)\right)\right)\right) g'(a) \geq 0$$

$$\iff g'(a) \leq G(\xi) = \frac{\frac{d}{da}\left(\bar{m}_k(a)\right)^2}{2\left(\lambda \frac{d}{da}\left(\bar{\sigma}_k^2(a)\right) + \theta \frac{d}{da}\left(\sigma_\mu^2\left(m_k(a)\right)\right)\right)}$$

Then,

$$\beta_k(a) = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\frac{d}{da}\left(\bar{m}_k(a)\right) + \sqrt{\frac{d}{da}\left(\bar{m}_k(a)\right)^2 - 2\left(\lambda \frac{d}{da}\left(\bar{\sigma}_k^2(a)\right) + \theta \frac{d}{da}\left(\sigma_\mu^2\left(m_k(a)\right)\right)\right) g'(a)}}{-\left(\lambda \frac{d}{da}\left(\bar{\sigma}_k^2(a)\right) + \theta \frac{d}{da}\left(\sigma_\mu^2\left(m_k(a)\right)\right)\right)}$$

$$= \frac{-\frac{d}{da}\left(\bar{m}_k(a)\right) + \sqrt{\frac{d}{da}\left(\bar{m}_k(a)\right)^2 - 2\left(\lambda \frac{d}{da}\left(\bar{\sigma}_k^2(a)\right) + \theta \frac{d}{da}\left(\sigma_\mu^2\left(m_k(a)\right)\right)\right) g'(a)}}{-\left(\lambda \frac{d}{da}\left(\bar{\sigma}_k^2(a)\right) + \theta \frac{d}{da}\left(\sigma_\mu^2\left(m_k(a)\right)\right)\right)} \frac{(A+B)}{(A+B)}$$

where:

$$A = -\frac{d}{da}\left(\bar{m}_k(a)\right), B = \sqrt{\frac{d}{da}\left(\bar{m}_k(a)\right)^2 - 2\left(\lambda \frac{d}{da}\left(\bar{\sigma}_k^2(a)\right) + \theta \frac{d}{da}\left(\sigma_\mu^2\left(m_k(a)\right)\right)\right) g'(a)}$$

Hence,

$$\beta_k(a) = \frac{\frac{d}{da}\left(\bar{m}_k(a)\right)^2 - \frac{d}{da}\left(\bar{m}_k(a)\right)^2 - 2\left(\lambda \frac{d}{da}\left(\bar{\sigma}_k^2(a)\right) + \theta \frac{d}{da}\left(\sigma_\mu^2\left(m_k(a)\right)\right)\right) g'(a)}{-\left(\lambda \frac{d}{da}\left(\bar{\sigma}_k^2(a)\right) + \theta \frac{d}{da}\left(\sigma_\mu^2\left(m_k(a)\right)\right)\right)(A+B)}$$

$$\beta_k(a) = \frac{2\, g'(a)}{\frac{d}{da}\left(\bar{m}_k(a)\right) + \sqrt{\frac{d}{da}\left(\bar{m}_k(a)\right)^2 - 2\left(\lambda \frac{d}{da}\left(\bar{\sigma}_k^2(a)\right) + \theta \frac{d}{da}\left(\sigma_\mu^2\left(m_k(a)\right)\right)\right) g'(a)}}$$

Proof of Theorem 1:

From Proposition 2, the principal's utility function is:

$$\Psi_k(a) = \bar{m}_x(a) - \frac{1}{2}\lambda\left(\beta_k \bar{\sigma}_k(a)\right)^2 - \frac{1}{2}\theta\left(\beta_k \sigma_\mu\left(m_k(a)\right)\right)^2 - g(a)$$

$$= \bar{m}_x(a) - \frac{1}{2}\lambda f_k^2(a) - \frac{1}{2}\theta y_k^2(a) - g(a)$$

Hence,

$$\frac{d}{dk}\left(\Psi_k(a)\right) = -\lambda f_k \frac{d}{dk}(f_k) - \theta y_k \frac{d}{dk}(y_k)$$

Similar to Feltham and Wu (2001), $\frac{d}{dk}(f_k) > 0$,

Note that, $y_k(a) = \beta_k(a)\left[\sum_{j=1}^{n} \mu_j \left(m_{kj}(a) - \bar{m}_k(a)\right)^2\right]^{\frac{1}{2}}$

Hence,

$$\frac{d}{dk}(y_k(a)) = \frac{d}{dk}(b_k(a))\left[\sum_{j=1}^{n} \mu_j \left(m_{kj}(a) - \bar{m}_k(a)\right)^2\right]^{\frac{1}{2}}$$

$$+\ \beta_k(a)\frac{1}{2}\left[\sum_{j=1}^{n} \mu_j \left(m_{kj}(a) - \bar{m}_k(a)\right)^2\right]^{-\frac{1}{2}}$$

$$\times\ \beta_k(a)\left[\sum_{j=1}^{n} 2\mu_j \left(m_{kj}(a) - \bar{m}_k(a)\right)\frac{d}{dk}\left(m_{kj}(a) - \bar{m}_k(a)\right)\right]$$

$$= \frac{d}{dk}(b_k(a))\left[\sum_{j=1}^{n} \mu_j \left(m_{kj}(a) - \bar{m}_k(a)\right)^2\right]^{\frac{1}{2}}$$

$$+ b_k(a)\frac{\sum_{j=1}^{n} \mu_j \left(m_{kj}(a) - \bar{m}_k(a)\right)\frac{d}{dk}\left(m_{kj}(a) - \bar{m}_k(a)\right)}{\left[\sum_{j=1}^{n} \mu_j \left(m_{kj}(a) - \bar{m}_k(a)\right)^2\right]^{\frac{1}{2}}}$$

Where,

$\frac{d}{dk}\left(b_k(a)\right)\left[\sum_{j=1}^{n} \mu_j \left(m_{kj}(a) - \bar{m}_k(a)\right)^2\right]^{\frac{1}{2}} > 0$

$b_k(a) > 0$

$\sum_{j=1}^{n} \mu_j \left(m_{kj}(a) - \bar{m}_k(a)\right)\frac{d}{dk}\left(m_{kj}(a) - \bar{m}_k(a)\right)$ is uncertain if < or > 0

$\left[\sum_{j=1}^{n} \mu_j \left(m_{kj}(a) - \bar{m}_k(a)\right)^2\right]^{\frac{1}{2}} > 0$