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Granger Causality among Graphs and Application to Functional Brain Connectivity in Autism Spectrum Disorder

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Abstract: Graphs/networks have become a powerful analytical approach for data modeling. Besides, with the advances in sensor technology, dynamic time-evolving data have become more common. In this context, one point of interest is a better understanding of the information flow within and between networks. Thus, we aim to infer Granger causality (G-causality) between networks' time series. In this case, the straightforward application of the well-established vector autoregressive model is not feasible. Consequently, we require a theoretical framework for modeling time-varying graphs. One possibility would be to consider a mathematical graph model with time-varying parameters (assumed to be random variables) that generates the network. Suppose we identify G-causality between the graph models' parameters. In that case, we could use it to define a G-causality between graphs. Here, we show that even if the model is unknown, the spectral radius is a reasonable estimate of some random graph model parameters. We illustrate our proposal's application to study the relationship between brain hemispheres of controls and children diagnosed with Autism Spectrum Disorder (ASD). We show that the G-causality intensity from the brain's right to the left hemisphere is different between ASD and controls.

Keywords: Granger causality; random graphs; spectral radius; brain connectivity; autism spectrum disorder

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1. Introduction

Graphs have been extensively used to model high-dimensional systems with complex dependence structures. Networks are ubiquitous, from genes [1,2] to social systems [3,4]. Besides, with the advances in sensor technology, dynamic, time-evolving data have become more frequently available [5,6]. In this context, time-series analysis methods on dynamic networks became relevant to understand how networks evolve and interact. For example, we would like to infer the information flow between networks.

Clive Granger introduced a causality concept to analyze the relationships and influences among macroeconomic time series [7]. Granger causality consists of the idea that a cause cannot ever occur after its effect. To identify Granger causality (G-causality) between two time series, e.g., stock markets, we may use the vector autoregressive (VAR) model [8]. It is a well-established method and widely used in economy [9,10] and biology [11,12]. The VAR model has many variants. For example, the Dynamic VAR identifies time-varying G-causality [13–15]. The Sparse VAR is helpful when the number of parameters is greater than the number of observations [16–18]. The Nonlinear VAR identifies nonlinear G-causality [19,20] in contrast to the classic VAR that identifies only linear relationships. The Structural VAR allows an explicit structure of the contemporaneous effects and constraints on the lagged effects [21–23]. There is also an approach based on the canonical correlation analysis to identify G-causality between sets of time series [12,24].