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Structure and causality relations in a global network of financial companies.

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Abstract

This work uses the stocks of the 197 largest companies in the world, in terms of market capitalization, in the financial area in the study of causal relationships between them using Transfer Entropy, which is calculated using the stocks of those companies and their counterparts lagged by one day. With this, we can assess which companies influence others according to sub-areas of the financial sector, which are banks, diversified financial services, savings and loans, insurance, private equity funds, real estate investment companies, and real estate trust funds. We also analyzed the causality relations between those stocks and the network formed by them based on this measure, verifying that they cluster mainly according to countries of origin, and then by industry and sub-industry. Then we collected data on the stocks of companies in the financial sector of some countries that are suffering the most with the current credit crisis: Greece, Cyprus, Ireland, Spain, Portugal, and Italy, and assess, also using transfer entropy, which companies from the largest 197 are most affected by the stocks of these countries in crisis. The intention is to map a network of influences that may be used in the study of possible contagions originating in those countries in financial crisis.

1 Introduction

In his speech delivered at the Financial Student Association in Amsterdam, in 2009, Andrew G. Haldane (2009), Executive Director of Financial Stability of the Bank of England, called for a rethinking of the financial network, that is the net formed by the connections between banks and other financial institutions. He warned that, in the last decades, this network had become more complex and less diverse, and that these facts may have led to the crisis of 2008.

According to him, it was the belief of theoreticians and practitioners of the financial market that connectivity between financial companies meant risk diversification and dispersion, but further studies showed that networks of certain complexity exhibit a robust but fragile structure, where crises may be dampened by sharing a shock among many institutions, but where they may also spread faster and further due to the connections between companies. Other issue to be considered was the fact that some nodes in the financial network were very connected to others, while some were less connected. The failure of a highly connected node could, thus, spread a small crisis to many other nodes in the network. Another factor was the small-world property of the financial network, where one company was not very far removed from another, through relations between common partners, or common partners of partners.

Such connected network was also more prone to panic, tightening of credit lines, and distress sales of assets, some of them caused by uncertainties about who was a counterpart to failing companies. Due to some financial innovations, risk was now shared among many parties, some of them not totally aware of all the details of a debt that was sectorized, with risk being decomposed and then reconstituted in packages that were then resold to other parties. This made it difficult to analyze the risk of individual institutions, whose liabilities were not completely known even to themselves, since they involved the risks of an increasingly large number of partners.

The other important aspect, the loss of diversity, increased when a large number of institutions adopted the same strategies in the pursuit of return and in the management of risk. Financial companies were using the same models and using the same financial instruments, with the same aims.

In the same speech, Haldane pointed at some directions that could improve the stability of the financial network. The first one was to map the network, what implied the collection, sharing and analysis of data.

This analysis needed to include techniques that didn't focus only on the individual firms, like most econometric techniques do, but also on the network itself, using network techniques developed for other fields, like ecology or epidemiology. The second was to use this knowledge to properly regulate this network. The third was to restructure the financial network, eliminating or reinforcing weak points. All these need a better understanding of the connections between financial institutions and how these connections influence the very topology of the financial network.

This article contributes to the first direction pointed by Haldane, that of understanding the international financial network. We do it by calculating two types of networks based on the daily returns of the stocks of the 197 largest financial companies across the world in terms of market capitalization that survive a liquidity filter. These include not just banks, but also diversified financial services, insurance companies, one investment company, a private equity, real estate companies, REITS (Real Estate Investment Trusts), and savings & loans institutions. We use the daily returns in order to build the networks because we believe that the price of a stock encodes a large amount of information about the company to which it is associated that goes beyond the information about the assets and liabilities of the company. Also, we believe that it is more interesting to study the effects of stock prices on other stock prices, as in the propagation of a financial crisis, rather than the spreading of defaults, since defaults are are events that are usually avoided by injecting external capital into banks.

The first network is built on the correlations between the log-returns of those equities, and the second one is built using Transfer Entropy, a measure first developed in information science. The first network is an undirected one, which expresses the common movements of stocks, and the second one is a directed one, which reveals causality relations between equities. Both networks are used in order to determine which are the most central nodes, according to diverse centrality criteria. We also enlarge the original network obtained by Transfer Entropy to include the most liquid stocks belonging to financial companies in some European countries that have been receiving much attention recently due to the fact that they are facing different degrees of economic crises, and determine who are the major financial companies in the world that are most affected by price movements of those stocks, and which of those stocks belonging to countries in crisis are the most influent ones.

There is an extensive literature on the propagation of shocks in networks of financial institutions, and describing all the published works in this subject is beyond the scope of this article. So, we shall here only cite the article that is considered the seminal work in networks of financial institutions and some review articles in the field. Most of the works in this field can be divided into theoretical and empirical ones, most of them considering networks of banks where the connections are built on the borrowing and lending between them. In most theoretical works, networks are built according to different topologies (random, small world, or scale-free), and the propagation of defaults is studied on them. The conclusions are that small world or scale-free networks are, in general, more robust to cascades (the propagation of shocks) than random networks, but they are also more prone to propagations of crises if the most central nodes (usually, the ones with more connections) are not themselves backed by sufficient funds. Most empirical works are also based on the structure derived from the borrowing and lending between banks, and they show that those networks exhibit a core-periphery structure, with few banks occupying central, more connected positions, and others populating a less connected neighbourhood. Those articles showed that this structure may also lead to cascades if the core banks are not sufficiently resistant, and that the network structures changed considerably after the crisis of 2008, with a reduction on the number of connected banks and a more robust topology against the propagation of shocks.

The work that is considered the first that deals with the subject is the one of Allen and Gale (2000), where the authors modeled financial contagion as an equilibrium phenomenon, and concluded that equilibrium is fragile, that liquidity shocks may spread through the network, and that cascade events depend on the completeness of the structure of interregional claims between banks. In their model, they used four different regions, which may be seen as groups of banks with some particular specializations. They focused in one channel of contagion, which is the overlapping claims that different regions or sectors of the banking system have on one another. According to them, another possible channel of contagion that is not being considered is incomplete information among agents. As an example, the information of a shock in one region may create a self-fulfilling shock in another region if that information is used as a prediction of shocks in other regions. Another possible channel of contagion is the effect of currency markets in the propagation of shocks from one country to another. In

their results, the spreading of a financial crisis depends crucially on the topology of the network. A completely connected network is able to absorb shocks more efficiently, and a network with strong connections limited to particular regions which are not themselves well connected is more prone to the dissemination of shocks.

Later, Allen and Babus (2009) made a review of the progress of the network approach to the propagation of crises in the financial market. They concluded that there is an urgent need for empirical work that maps the financial network, so that the modern financial systems may be better understood, and that a network perspective would not only account for the various connections within the financial sector or between the financial sector and other sectors, but also would consider the quality of such links. Upper (2011) made a survey of a diversity of simulation methods that have been used with a variety of financial data in order to study contagion in financial networks, and made a comparison between the various methods used.

This article is organized as follows. Section 2 explains the data used in the article and some of the methodology. Section 3 uses the correlations between stocks in order to exemplify some of the techniques to be used for Transfer Entropy, but yet in a more familiar background. Section 4 explains Transfer Entropy and uses it in order to study the causality relations between the stocks of financial institutions. That section also highlights which are the most central stocks according to different centralities criteria. Section 5 studies the relationships between countries in crisis in Europe with the largest financial institutions, analyzing which stocks are more affected by movements in the stocks belonging to those countries in crisis. Finally, Section 6 shows some conclusions and possible future work.

2 Data and methodology

In order to choose appropriate time series of the top stocks in terms of market capitalization belonging to the financial sector, we used the S&P 1200 Global Index, which is a free-float weighted stock market index of stocks belonging to 31 countries. The stocks belonging to the index are responsible for approximately 70 percent of the total world stock market capitalization, and 200 of them belong to the financial sector, as classified by Bloomberg. From those, we extracted 197 stocks that had enough liquidity with respect to the working days of the New York Stock Exchange (NYSE). From the 197 stocks, 79 belong to the USA, 10 to Canada, 1 to Chile, 21 to the UK, 4 to France, 5 to Germany, 7 to Switzerland, 1 to Austria, 2 to the Netherlands, 2 to Belgium, 5 to Sweden, 1 to Denmark, 1 to Finland, 1 to Norway, 6 to Italy, 4 to Spain, 1 to Portugal, 1 to Greece, 12 to Japan, 9 to Hong Kong, 1 to South Korea, 1 to Taiwan, 3 to Singapore, and 18 to Australia. The stocks and their classification according to industry and sub-industry are listed in Appendix A.

We took the daily closing prices of each stock, and the resulting time series of all 197 stocks were compared with the time series of the NYSE, which was taken as a benchmark, since it is by far the major stock exchange in the world. If an element of the time series of a stock occurred for a day in which the NYSE wasn't opened, then this element was deleted from the time series, and if an element of the time series of a stock did not occur in a day in which the NYSE functioned, then we repeated the closing price of the previous day. The idea was not to eliminate too many days of the time series by, as an example, deleting all closing prices in a day one of the stock exchanges did not operate. The methodology which we chose would be particularly bad for stocks belonging to countries where weekends occur on different days than for Western countries, like Muslim countries or Israel, but since no stocks from our set belong to those countries, differences on weekends are not relevant here.

The data are organized so as to place stocks of the same country together, and then to discriminate stocks by industry and subindustry, according to the classification used by Bloomberg. From the 197 stocks, 80 belong to Banks, 27 to Diversified Financial Services, 50 to Insurance Companies, 1 to an Investment Company, 1 to a Private Equity, 8 to Real Estate Companies, 28 are REITS (Real Estate Investment Trusts), and 2 belong to Savings & Loans.

In order to reduce non-stationarity of the time series of the daily closing prices, we consider the log-returns of the closing prices, defined as

$$R_t = \ln(P_t) - \ln(P_{t-1}) \ , \tag{1}$$

where P_t is the closing price of the stock at day t and P_{t-1} is the closing price of the same stock at day $t-1$.

Since the stocks being considered belong to stock markets that do not operate at the same times, we run into

the issue of lagging or not some stocks. Sandoval (2012a), when dealing with stock market indices belonging to stock markets across the globe, showed that it is not very clear that an index has to be lagged with respect to another, except in cases like Japan and the USA. A solution is to use both original and lagged indices in the same framework, and to do all calculations as if the lagged indices were different ones. The same procedure is going to be followed here with the log-returns of the closing prices of the stocks that have been selected, so we shall deal with $2 \times 197 = 394$ time series.

3 Correlations

Our first analysis of the data is based on the familiar correlation structure between the stocks. Correlation will be used in order to establish some methodology that will be followed later on for Transfer Entropy. The time series of log-returns of the 197 stocks with largest stock market capitalization and their lagged counterparts are used in order to calculate a correlation matrix C whose elements C_{ij} are the correlations between stocks i and j. We use the usual Pearson correlation in this calculation, since previous results obtained with this type of (linear) correlation are in good accordance with results obtained using the Spearman rank correlation (Sandoval, 2013), which is more complex to calculate.

The structure of the resulting correlation matrix may be visualized in Figure 1a, where we plot a false color map of the elements of the correlation matrix, with lighter colors denoting higher correlations and darker ones denoting lower correlations. The figure displays the correlations in such a way that the leftmost and lowest corner corresponds to the correlation between element 1 with itself. The number of each stock grows from left to right and from the bottom to the top. The same configuration will be used in all other representations of matrices in this article. As expected, the diagonal elements are the brightest ones, with correlation 1 between all stocks and themselves. It is also possible to identify some clusters. First of all, there is a repetition pattern of stocks 1 to 197 and 198 to 394, corresponding to the original log-returns and the lagged ones in Figure 1a. If one plots the correlation matrix obtained by considering the original log-returns, plus lagged log-returns by one and by two days, the same structure repeats itself twice.

We may also identify other blocks, related with geographical position. The first one, going from 1 to 90, corresponds to stocks from North America (USA and Canada); the second one, from 91 to 152, corresponds to European stocks, plus Chile, which corresponds to a darker shade at 91 and which is here closer to Europe than to America; the third one is a loose structure of Australasian stocks, from 153 to 196. As said before, the pattern repeats itself for the lagged stocks. There are clear relations among these three main clusters, as shown by the brighter regions around the American and the European blocks, and also around the Australasian block. We may also see interaction of the Australasian block with the lagged stocks from America and Europe, showing a relation between Western stock markets with the Asian stock markets of the next day.

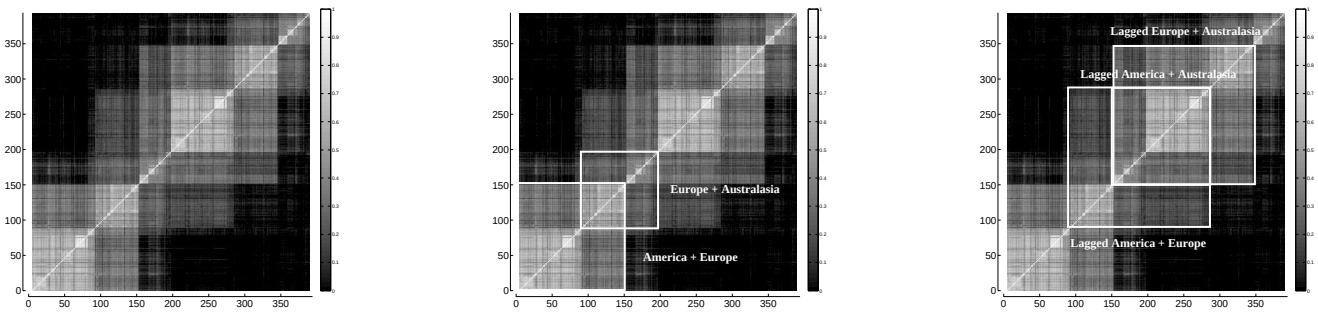

Figure 1a Figure 1b Figure 1c

Fig. 1. False color graph of the correlation matrix involving original and lagged stocks. Figure 1a shows the correlation matrix, Figure 1b highlights the correlations between stocks on the same day, and Figure 1c highlights the correlations between original and lagged stocks.

In Figure 1b, we highlighted the correlations between American and European stocks in the same day, and the correlations between European and Australasian stocks in the same day. Note that there are also weaker correlations between stocks of Australasia and stocks of America on the same day. In Figure 1c, we outline

the correlations of the lagged stocks of America with the current stocks of Europe and of Australasia, and the correlations of lagged European stocks with the current stocks of Australasia. The figures show relations between stocks in the same day and between stocks of a previous day with stocks of the next day.

Within each main block, there are also some concentrations of brighter spots. This is more clearly visible if one plots only the elements of the correlation matrix that are above a certain correlation threshold. In Figure 2, we plot these values for the correlation matrix in black against a white background for threshold values 0.8, 0.7, 0.6, 0.5, 0.4, and 0.3, respectively. For 0.8, we find a structure of highly correlated stocks corresponding to REITS negotiated at the NYSE (original and lagged), between numbers 63 and 79. Between numbers 1 and 15, there is a loose agglomeration of Diversified Banking Institutions and Super-Regional-Banks of the USA. For threshold 0.7, there is a loose cluster of Banks from the USA (1 to 22), a tight cluster of Diversified Financial Services (Credit Card, Investment Management and Advisory Services) (27 to 33), a loose cluster of Insurance Companies (Multi-line, Life/Health, and Property/Casualty) (43 to 58), the cluster of REITS (63 to 79), a tight cluster of Canadian Commercial Banks (80 to 85), a small, but tight cluster of stocks from Life/Health Insurance Companies negotiated at the London Stock Exchange (102 to 105), another tight cluster of REITS negotiated at the UK (108 to 111), a tight cluster of stocks negotiated at the Paris Stock Exchange (112 to 115), immersed in a loose cluster of stocks negotiated in Germany, and of Banks from Switzerland (112 to 122), a tight cluster of stocks negotiated in Sweden (133 to 137), a cluster of stocks negotiated in Italy and Spain (141 to 150), a cluster of stocks negotiated in Japan (Banks and Diversified Financial Services) (154 to 160), a tight cluster of Real Estate companies from Japan (163 to 165), a cluster of stocks from Banks, Diversified Financial Services and Insurance from Hong Kong (167 to 172), a cluster made of a pair of stocks from Real Estate companies from Hong Kong (173 and 174), a cluster of Commercial Banks from Singapore (177 to 179), and a cluster of Commercial Banks from Australia. We also find some strong interactions between stocks from Italy and Spain with stocks from France, Germany, and Switzerland. The same pattern is followed by the lagged stocks.

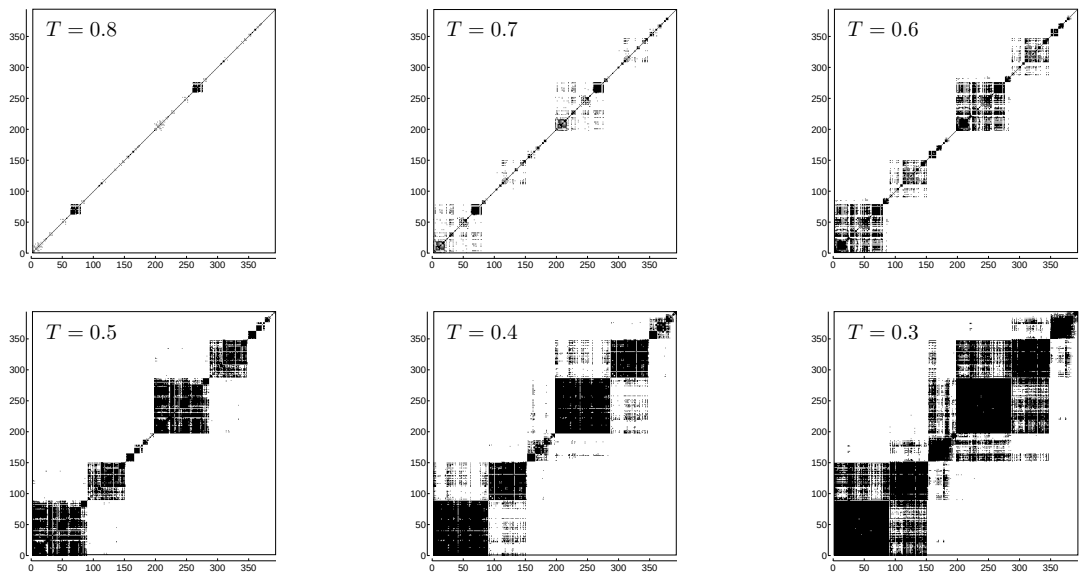

Fig. 2. Elements of the correlation matrix that are above some threshold. Points above the threshold are represented in black, and points bellow it are represented in white. The figures correspond to the following thresholds: 0.8, 0.7, 0.6, 0.5, 0.4, and 0.3, respectively.

For threshold 0.6, we already find some macro structures, like a cluster of stocks negotiated in the USA (1 to 79), interacting weakly with a cluster of stocks from Canada (80 to 89), a looser cluster of stocks from European countries (91 to 150), a cluster of stocks from Japan (153 to 165), a cluster of stocks from Hong Kong (166 to 174), a cluster of stocks from Singapore (177 to 179), and a cluster of Commercial Banks, one of Diversified Financial Services, and one of Insurance from Australia (180 to 184, and 186). For threshold 0.5, the macro structures according to continents become clearer for America and Europe, but Australasia is still fragmented. For 0.4, we start to see the correlations between the American and the European stocks,

and an Australasian cluster becomes visible. We may also see a strong connection of an insurance company from Japan (number 162) with the lagged stocks of the USA and Canada. For threshold 0.3, the connections between America and Europe are strengthened, and we may also see connections of Australasia with European stocks of the same day, and with North American and European stocks of the previous day.

3.1 Asset graphs

Another way to analyze the structure of the correlations among the stocks of the financial sector here studied is to use asset graphs, which are based on a proper distance measure derived from the correlation matrix and on a threshold value for this distance, as in the works of Onnela, Chakraborti, Kaski, and Kertész (2002), Onnela, Chakraborti, and Kaski (2003), Onnela, Chakraborti, Kaski, and Kertész (2003a), Onnela, Chakraborti, Kaski, and Kertész (2003b), Onnela, Kaski, and Kertész (2003), Sinha and Pan (2007), Ausloos and Lambiotte (2007), Sandoval (2012b), and Sandoval (2013). In an asset graph, given a certain threshold value, all distances below this threshold are represented as edges (links) between nodes, and all nodes without edges are not represented. This is a way of filtering some of the information and noise contained in a correlation matrix.

There are many ways to define a distance measure based on a correlation matrix, but the most used one in applications to financial markets is given by Mantegna (1999):

$$d_{ij} = \sqrt{2\left(1 - c_{ij}\right)}\ , \tag{2}$$

where c_{ij} is the correlation between nodes i and j. As correlations between stocks vary from -1 (anticorrelated) to 1 (completely correlated), the distance between them vary from 0 (totally correlated) to 2 (completely anticorrelated). Totally uncorrelated stocks would have distance 1 between them.

Based on the distance measures, m-dimensional coordinates are assigned to each stock using an algorithm called Classical Multidimensional Scaling (Borg and Groener, 2005), which is based on minimizing the stress function

$$S = \left[\frac{\sum_{i=1}^{n}\sum_{j>i}^{n}\left(\delta_{ij} - \bar{d}_{ij}\right)^2}{\sum_{i=1}^{n}\sum_{j>i}^{n} d_{ij}^2}\right]^{1/2} \quad , \quad \bar{d}_{ij} = \left[\sum_{a=1}^{m}\left(x_{ia} - x_{ja}\right)^2\right]^{1/2} \ . \tag{3}$$

where δ_{ij} is 1 for $i = j$ and zero otherwise, n is the number of rows of the correlation matrix, and $\bar{d}_{ij}$ is an m-dimensional Euclidean distance (which may be another type of distance for other types of multidimensional scaling). The outputs of this optimization problem are the coordinates x_{ia} of each of the nodes, where $i = 1, \cdots, n$ is the number of nodes and $a = 1, \cdots, m$ is the number of dimensions in an m-dimensional space. The true distances are only perfectly representable in $m = n$ dimensions, but it is possible for a network to be well represented in smaller dimensions. In the case of this article we shall consider $m = 2$ for a 2-dimensional visualization of the network, being the choice a compromise between fidelity to the original distances and the easiness of representing the networks in a two dimensional medium.

Figure 3a shows the stocks represented as nodes at the coordinates calculated by this procedure. White dots stand for the original log-returns, and black dots for their lagged values. There is a clear division between original and lagged stocks. Figure 3b represents the continents to which the stocks (original and lagged) belong, showing a clear division according to geography. The colors are black for America, white for Europe, and gray for Australasia. Note that the present stocks from Australasia are close both to the lagged stocks from America and to present stocks from Europe.

Figure 3 does not correspond to a network, since there are no edges between the nodes. By using the concept of asset graph, we may choose values for a distance threshold and represent only the edges that are below this threshold and the nodes connected by them. By choosing appropriate threshold values for the distance, above which edges and nodes are removed, we may obtain some filtered representations of the correlation structure between the stocks. Figure 4 presents the asset graphs for thresholds 0.6 and 0.8, in which we may see the formation of structures between the nodes. We did not represent the asset graphs for lower values than 0.6,

because there are too few connections for them, nor higher values than 0.8, because the number of edges is so large that the pictures become hard to understand.

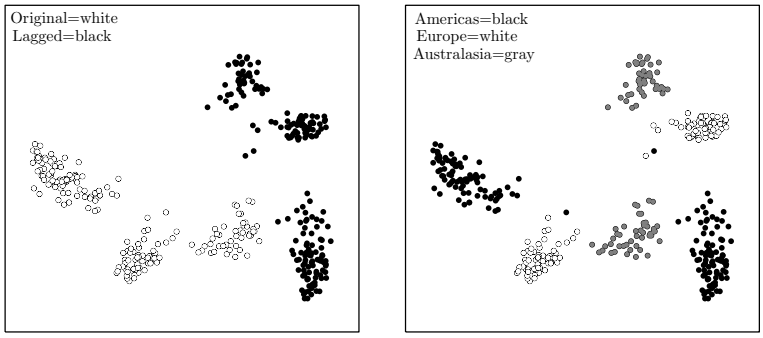

Figure 3a Figure 3b

Fig. 3. Two dimensional representation of the stocks as nodes in coordinates that simulate the distances between them. In Figure 3a, white dots represent the original log-returns, and black dots represent their lagged values by one day. In Figure 3b, continents are highlighted: stocks belonging to America are represented as white dots, stocks belonging to Europe are represented as black dots, and stocks belonging to Eurasia are represented by gray dots.

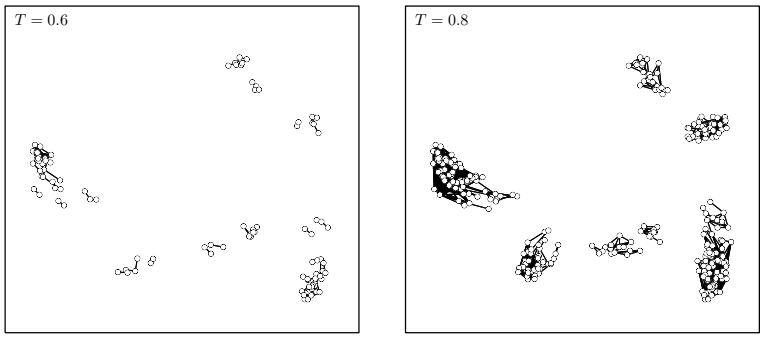

Figure 4a Figure 4b

Fig. 4. Asset graphs for the stocks (original and lagged ones) at thresholds 0.6 and 0.8.

For threshold 0.4, the only connections are between the pairs Banco Bradesco and Itau Unibanco Holding (both stocks of Brazilian banks negotiated at the NYSE), Boston Properties (REITS - Office Porperty) and Vornado Realty Trust (REITS - Diversified), both from the USA, and Banco Bilbao Vizcaya Argentaria and Banco Santander (both commercial banks from Spain). For threshold 0.6, we have a large cluster comprised of REITS (Real Estate Investment Trusts), whose members are Apartment Investment & Management, Avalon Bay Communities, Equity Residential (Apartments), Boston Properties (Office Property), Host Hotels & Resorts (Hotels), Prologis (Warehouse/Industrial), Public Storage (Storage), Simon Property Group (Regional Malls), Kimco Realty (Shopping Centers), Ventas, HCP, Health Care REIT (Health Care), Vornado Realty Trust, and Plum Creek Timber (Diversified). There is also a small cluster of banks from the USA, comprised of stocks of Bank of America, JP Morgan Chase, US Bancorp, and Wells Fargo, and the pairs The Goldman Sachs Group and Morgan Stanley Banks (Diversified Banking Institutions), Comerica and BB&T (Commercial Banks), The Bank of New York Mellon and Northern Trust (Fiduciary Banks), Franklin Resources and T Rowe Price Group (Investment Management / Advisory Services), Principal Financial Group and Prudential Financial (Life/Health Insurance), and The Travelers Cos and The Chubb (Property/Casualty Insurance). There is a small cluster of Canadian stocks, comprised of stocks of the Bank of Nova Scotia, the Royal Bank of Canada, and The Toronto-Dominion Bank, the pair of British REITS British Land and Land Securities Group, the small cluster of French banks, comprised of Crédit Agricole, BNP Paribas, and Société Générale, a small cluster of Japanes banks, Shinsei Bank, Sumitomo Mitsui Financial Group, and Mizuho Financial Group, the Japanese investment banks Daiwa Securities Group and Nomura Holdings, the Japanese real estate companies Mitsui Fudosan, Mitsubishi Estate, and Sumitomo Realty & Development, the pair of banks from Hong Kong

Industrial & Commercial Bank of China and China Construction Bank Corp, and the pair of real estate companies from Hong Kong Cheung Kong Holdings and Sun Hung Kai Properties. The pairs Bradesco and Itau Unibanco, and Bilbao Vizcaya and Santander are still isolated from the other nodes.

For threshold 0.8, there is a large cluster of stocks of the USA, a cluster of stocks from Canada, a cluster of three banks of the UK, a cluster of four REITS from the UK, and a mixed cluster of European stocks. There are also four more clusters, one of Japanese stocks, another of Hong Kong stocks, a cluster of stocks of Singapore, and a cluster of stocks of Australia. For higher thresholds, the individual clusters merge more often, beginning with North America and Europe, and with the merging of the Australasian stocks, and then between the two main blocks and across original and lagged stocks. Figure 2 and the discussion associated with it is a good way to visualize the clustering that occurs here, since distance and correlation are related, although in a nonlinear way.

Some of the information obtained from the correlation matrix is plagued by noise, which may originate from the finiteness of data, residual non-stationarity of the time series, and many other sources. In order to gauge the effect of noise in the asset graphs, we calculated randomized time series for each stock, in which the order of the elements of each time series was randomly shuffled, so as to destroy any possible correlation between each time series but to preserve the frequency distribution of each one. We simulated 1,000 correlation matrices based on such shuffled time series, and calculated the distance matrix for each one. Excluding distances equal to zero, which is the distance between a stock and itself, we obtained a minimum distance equal to $d_{min} = 1.265 \pm 0.003$ (average $\pm$ standard deviation). It has been shown empirically by Sandoval (2013) that we may obtain more information about an asset graph if we consider thresholds that are close to this lower limit for noise. So, we shall consider the network of nodes whose edges are below the distance $d = 1.2$.

3.2 Centrality

In network theory, the centrality of a node is important in the study of which nodes are, by some standard, more influential than others. Such measures may ne used, for instance, in the study of the propagation of epidemics, or the propagation of news, or, in the case of stocks, in the spreading of high volatility. There are various centrality measures (Newman, 2010), tending to different aspects of what we may think of "central". For undirected networks, for instance, we have Node Degree (ND), which is the total number of edges between a node and all others to which it is connected. This measure is better adapted to asset graphs, where not all nodes are connected between them, and varies according to the choice of threshold, as in Sandoval (2013). Another measure than can be used for asset graphs is Eigenvector Centrality (EC), which takes into account not just how many connections a node has, but also if it is localized in a region of highly connected nodes. There is also a measure called Closeness Centrality (CC) that measures the average distance (in terms of number of edges necessary to reach another node) of a certain node. This measure is larger for less central nodes, and if one wants a measure that, like the others, is larger for more central nodes, like the others we cited, then one may use Harmonic Closeness (HC), that is built on the same principles as Closeness Centrality, but is calculated using the inverse of the distances from one node to all others. The Betweenness Centrality (BC) of a node is another type of measure, that calculates how often a certain node is in the smaller paths between all other nodes. Still another measure of centrality, called Node Strength (NS), works for fully connected networks, and so is independent of thresholds in asset graphs, and takes into account the strength of the connections, which, in our case, are the correlations between the nodes. It measures the sum of the correlations of a node with all the others.

In Table 1, we present the nodes with highest centrality measures (top 5 values) by their names, countries they belong to, their industries and sub-industries. Names with an asterisk are lagged stocks. First, we find a preponderance of lagged stocks, except for Betweenness Centrality, since they are connected both among themselves and with the next days' values of stocks. There is also a preponderance of large banks (either listed as Diversified Banking Institutions or as Commercial Banks), with an important participation of Investment Management and Advisory Services for Node Degree and Eigenvector Centrality and a minor participation of Insurance companies. The USA dominates the scenario for Eigenvector centrality, since it has more stocks than the others, and they are more internally connected among themselves. Japan assumes preponderance in Betweenness centrality because of its role of connecting the lagged stocks of the USA and of Europe with the

next day stocks of both continents. Node Strength has results that point mostly at European stocks, which have larger correlation values among themselves, in average.

Centrality	Company	Country	Industry	Sub-Industry
Node Degree				
189	Credit Suisse Group*	France	Banks	Diversified Banking Institution
188	Franklin Resources*	USA	Diversif. Fin. Services	Investment Manag. / Adv. Services
187	Invesco*	USA	Diversif. Fin. Services	Investment Manag. / Adv. Services
185	T Rowe Price Group*	USA	Diversif. Fin. Services	Investment Manag. / Adv. Services
185	Zurich Insurance Group*	Switzerland	Insurance	Multi-line Insurance
Eigenvector				
0.089	Franklin Resources*	USA	Diversif. Fin. Services	Investment Manag. / Adv. Services
0.089	Invesco*	USA	Diversif. Fin. Services	Investment Manag. / Adv. Services
0.089	T Rowe Price Group*	USA	Diversif. Fin. Services	Investment Manag. / Adv. Services
0.089	Citigroup*	USA	Banks	Diversified Banking Institution
0.088	Itau Unibanco Holding*	USA	Banks	Commercial Bank
0.088	Prudential Financial*	USA	Insurance	Life/Health Insurance
0.088	Legg Mason*	USA	Diversif. Fin. Services	Investment Manag. / Adv. Services
0.088	The Goldman Sachs Group*	USA	Banks	Diversified Banking Institution
0.088	Ameriprise Financial*	USA	Diversif. Fin. Services	Investment Manag. / Adv. Services
0.088	Principal Financial Group*	USA	Insurance	Life/Health Insurance
Harmonic Closeness				
288	Franklin Resources*	USA	Diversif. Fin. Services	Investment Manag. / Adv. Services
286	Mitsubishi UFJ Financial Group	Japan	Banks	Diversified Banking Institution
283	Citigroup*	USA	Banks	Diversified Banking Institution
282.33	JP Morgan Chase & Co*	USA	Banks	Diversified Banking Institution
282.17	Bank of America*	USA	Banks	Diversified Banking Institution
Betweenness				
3908	Mitsubishi UFJ Financial Group	Japan	Banks	Diversified Banking Institution
2272	Zurich Insurance Group	Switzerland	Insurance	Multi-line Insurance
2126	Credit Suisse Group	France	Banks	Diversified Banking Institution
1970	National Australia Bank	Australia	Banks	Commercial Bank
1819	Mitsubishi Estate Co	Japan	Real Estate	Real Estate Mang/Serv.
Node Strength				
108.00	Deutsche Bank	Germany	Banks	Diversified Banking Institution
107.85	Franklin Resources*	USA	Diversif. Fin. Services	Investment Manag. / Adv. Services
106.52	Zurich Insurance Group	Switzerland	Insurance	Multi-line Insurance
105.20	Credit Suisse Group	France	Banks	Diversified Banking Institution
105.09	Allianz	Germany	Insurance	Multi-line Insurance

Table 1. Classification of stocks with highest centrality measures, the countries they belong to, their industry and sub-industry classifications, for threshold 1.2. Only the five stocks with highest centrality values are shown (more, in case of draws). The names with an asterisk are lagged stocks.

4 Transfer Entropy

Although useful for determining which stocks behave similarly to others, the correlations between them cannot establish a relation of causality or of influence, since the action of a stock on another is not necessarily symmetric. A measure that has been used in a variety of fields, and which is both dynamic and non-symmetric, is *Transfer Entropy*, developed by Schreiber (2000), which is based on the concept of *Shannon Entropy*, first developed in the theory of information by Shannon (1948). Transfer entropy has been used in the study of cellular automata in Computer Science, in the study of the neural cortex of the brain, in the study of social networks, in Statistics, and also in the analysis of financial markets, as in the works of Kwon and Yang (2008a), Kwon and Yang (2008b), and Jizba, Kleinert, and Shefaat (2012), Baek, and Dimp, Huergo, and Peter (2012).

In this section, we shall describe the concept of Transfer Entropy (TE), using it to analyze the data concerning the 197 stocks of companies of the financial sector and their lagged counterparts. We will start by describing briefly the concept of Shannon entropy.

4.1 Shannon Entropy

The American mathematician, electronic engineer and cryptographer, Claude Elwood Shannon (1916–2001), founded the theory of information in his work "A Mathematical Theory of Communication" (Shannon, 1948), in which he derived what is now known as the *Shannon entropy*. According to Shannon, the main problem of

information theory is how to reproduce at one point a message sent from another point. If one considers a set of possible events whose probabilities of occurrence are p_i, $i = 1, \cdots, n$, then a measure $H(p_1, p_2, \cdots, p_n)$ of the uncertainty of the outcome of an event given such distribution of probabilities should have the following three properties:

• $H(p_i)$ should be continuous in p_i;
• if all probabilities are equal, what means that $p_i = 1/n$, then H should be a monotonically increasing function of n (if there are more choices of events, then the uncertainty about one outcome should increase);
• if a choice is broken down into other choices, with probabilities c_j, $j = 1, \cdots, k$, then $H = \sum_{j=1}^{k} c_j H_k$, where H_k is the value of the function H for each choice.

Shannon proved that the only function that satisfies all three properties is given by

$$H = -\sum_{i=1}^{N} p_i \log_2 p_i \ , \tag{4}$$

where the sum is over all states for which $p_i \neq 0$ (Shannon's definition had a constant k multiplied by it, which has been removed here). The base 2 for the logarithm is chosen so that the measure is given in terms of bits of information. As an example, a device with two positions (like a flip-flop circuit) can store one bit of information. The number of possible states for N such devices would then be 2^N, and $\log_2 2^N = N$, meaning that N such devices can store N bits of information, as should be expected. This definition bears a lot of resemblance to Gibbs' entropy, but is more general, as it can be applied to any system that carries information.

The Shannon entropy represents the average uncertainty about measures i of a variable X (in bits), and quantifies the average number of bits needed to encode the variable X. In the present work, given the time series of the log-returns of a stock, ranging over a certain interval of values, one may divide such possible values into N different bins and then calculate the probabilities of each state i, what is the number of values of X that fall into bin i divided by the total number of values of X in the time series. The Shannon entropy thus calculated will depend on the number of bins that are selected. After selecting the number of bins, one associates a symbol (generally a number) to each bin.

Using the stocks of the J.P. Morgan (code JPM), classified as a Diversified Banking Institution, we shall give an example of the calculation of the Shannon Entropy for two different choices of bins. In Figure 5, we show the frequency distributions of the log-returns for the stocks of the J.P. Morgan from 2007 to 2012, which varied from -0.2323 to 0.2239 during that period, with two different binning choices. The first choice results in 24 bins of size 0.02, and the second choice results in 6 bins of size 0.1.

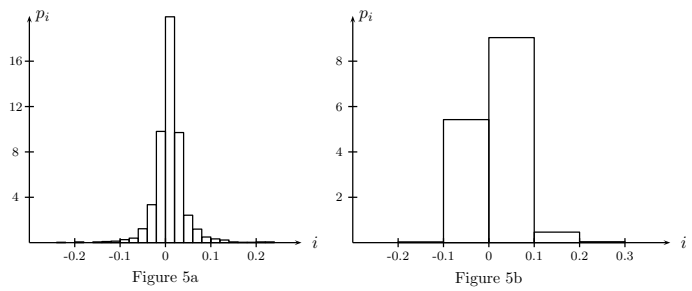

Fig. 5. Histograms of the log-returns of the stocks of the J.P. Morgan for two different binnings. In Figure 5a, we have 24 bins in intervals of size 0.02, and in Figure 5b, 6 bins in intervals of size 0.1.

To each bin is assigned a symbol, which, in our case, is a number, from 1 to 24 in the first case and from 1 to 6 in the second case. Figure 6 shows the assigning of symbols for the two choices of binning for the first log-returns of the stocks of the J.P. Morgan. Then, we calculate the probability that a symbol appears in the time series and then use (4) in order to calculate the Shannon entropy, which, in our case, is $H = 2.55$ for bins of size 0.02 and $H = 0.59$ for bins of size 0.1. The second result is smaller than the first one because there is less information for the second choice of binning due to the smaller number of possible states of the system. The difference in values, though, is not important, since we shall use the Shannon entropy as a means of comparing the amount of information in different time series.

Date	Log-return	Symbol
01/03/2007	−0.0048	12
01/04/2007	0.0025	13
01/05/2007	−0.0083	12
01/08/2007	0.0033	13
01/09/2007	−0.0042	12
01/10/2007	0.0073	13
⋮	⋮	⋮

Date	Log-return	Symbol
01/03/2007	−0.0048	3
01/04/2007	0.0025	4
01/05/2007	−0.0083	3
01/08/2007	0.0033	4
01/09/2007	−0.0042	3
01/10/2007	0.0073	4
⋮	⋮	⋮

Fig. 6. The assigning of symbols to the first values of the log-returns of the J.P. Morgan according to binning. On the left, for 24 bins and, on the right, for 6 bins.

Figure 7 shows the Shannon Entropy calculated for each stock in this study (the lagged stocks are not represented, since their entropies are nearly the same as the entropies of the original stocks). The results for both choices of binning are in fact very similar, and their correlation is 0.97. Stocks with higher Shannon Entropy are the most volatile ones. As one can see, the second choice, with larger bin sizes, shows the differences more sharply, which is one of the reasons why larger binnings are usually favored in the literature.

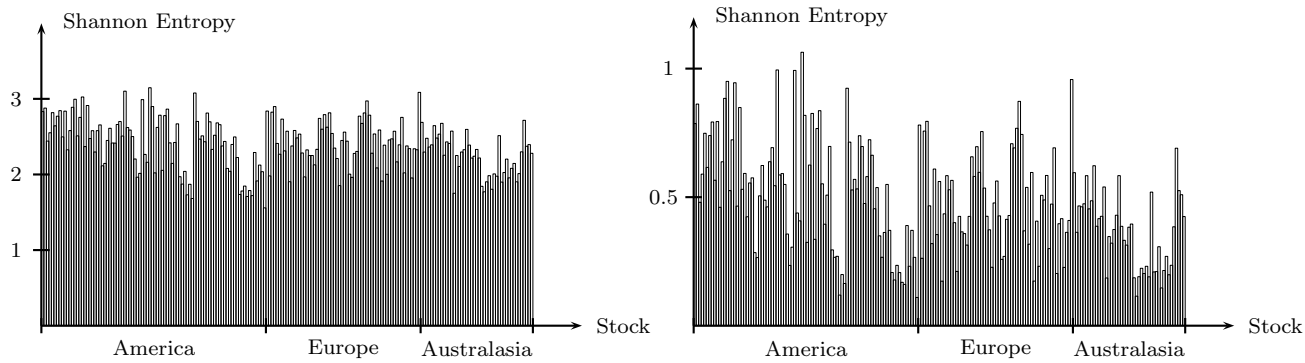

Figure 7a Figure 7b

Fig. 7. Shannon entropies of the 197 stocks, in the same order as they appear in the correlation matrix. Figure 7a is the Shannon Entropy for bins of size 0.02, and Figure 7b is the Shannon Entropy for bins of size 0.1.

4.2 Transfer Entropy

When one deals with variables that interact with one another, then the time series of one variable Y may influence the time series of another variable X in a future time. We may assume that the time series of X is a Markov process of degree k, what means that a state i_{n+1} of X depends on the k previous states of the same variable. This may be made more mathematically rigorous by defining that the time series of X is a Markov state of degree k if

$$p\left(i_{n+1}|i_n, i_{n-1}, \cdots, i_0\right) = p\left(i_{n+1}|i_n, i_{n-1}, \cdots, i_{n-k+1}\right) \ , \tag{5}$$

where $p(A|B)$ is the conditional probability of A given B, defined as

$$p(A|B) = \frac{p(A,B)}{p(B)} \ . \tag{6}$$

What expression (5) means is that the conditional probability of state i_{n+1} of variable X on all its previous states is the same as the conditional probability of i_{n+1} on its k previous states, meaning that it does not depend on states previous to the kth previous states of the same variable.

One may also assume that state i_{n+1} of variable X depends on the ℓ previous states of variable Y. The concept is represented in Figure 8, where the time series of a variable X, with states i_n, and the time series of a variable Y, with states j_n, are identified.

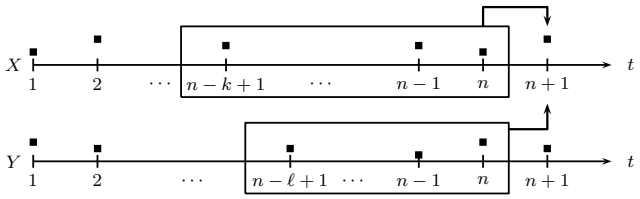

Fig. 8. Schematic representation of the transfer entropy $T_{Y\to X}$.

We may now define the concept of Transfer Entropy from a time series Y to a times series X as the average information contained in the source Y about the next state of the destination X that was not already contained in the destination's past. We assume that element i_{n+1} of the time series of variable X is influenced by the k previous states of the same variable and by the ℓ previous states of variable Y. The values of k and ℓ may vary, according to the data that is being used, and to the way one wishes to analyze the transfer of entropy of one variable to the other.

Transfer Entropy from variable Y to variable X is defined as

$$\begin{aligned} TE_{Y\to X}(k,\ell) &= \sum_{i_{n+1},i_n^{(k)},j_n^{(\ell)}} p\left(i_{n+1},i_n^{(k)},j_n^{(\ell)}\right)\log_2 p\left(i_{n+1}|i_n^{(k)},j_n^{(\ell)}\right) \\ &\quad - \sum_{i_{n+1},i_n^{(k)},j_n^{(\ell)}} p\left(i_{n+1},i_n^{(k)},j_n^{(\ell)}\right)\log_2 p\left(i_{n+1}|i_n^{(k)}\right) \\ &= \sum_{i_{n+1},i_n^{(k)},j_n^{(\ell)}} p\left(i_{n+1},i_n^{(k)},j_n^{(\ell)}\right)\log_2 \frac{p\left(i_{n+1}|i_n^{(k)},j_n^{(\ell)}\right)}{p\left(i_{n+1}|i_n^{(k)}\right)} , \end{aligned} \tag{7}$$

where i_n is element n of the time series of variable X and j_n is element n of the time series of variable Y, $p(A,B)$ is the joint probability of A and B, and

$$p\left(i_{n+1},i_n^{(k)},j_n^{(\ell)}\right) = p\left(i_{n+1},i_n,\cdots,i_{n-k+1},j_n,\cdots,j_{n-\ell+1}\right) \tag{8}$$

is the joint probability distribution of state i_{n+1}, of state i_n and its k predecessors, and the ℓ predecessors of state j_n, as in Figure 8.

This definition of Transfer Entropy assumes that events on a certain day may be influenced by events of k and ℓ previous days. We shall assume, with some backing from empirical data for financial markets, that only the day before is important. By doing so, formula (7) for the Transfer Entropy of Y to X becomes simpler:

$$TE_{Y\to X} = \sum_{i_{n+1},i_n,j_n} p\left(i_{n+1},i_n,j_n\right)\log_2 \frac{p\left(i_{n+1}|i_n,j_n\right)}{p\left(i_{n+1}|i_n\right)} = \sum_{i_{n+1},i_n,j_n} p\left(i_{n+1},i_n,j_n\right)\log_2 \frac{p\left(i_{n+1},i_n,j_n\right)p\left(i_n\right)}{p\left(i_{n+1},i_n\right)p\left(i_n,j_n\right)} , \tag{9}$$

where we took $k=\ell=1$, meaning we are using lagged time series of one day, only.

In order to exemplify the calculation of Transfer Entropy, we will now show some steps for the calculation of the Transfer Entropy from the Deutsche Bank to the J.P. Morgan. In Figure 9, first table, we show the initial part of the time series for the log-returns of the J.P. Morgan, which we call vector X_{n+1} (first column), for its values lagged by one day, vector X_n (second column), and the log-returns of the Deutsche Bank lagged by one day, vector Y_n (third column). Calculating the minimum and maximum returns of the entire set of time series, we obtain a minimum value $m=-1.4949$ and a maximum value $M=0.7049$. Considering then an interval $[-1.5,0.8]$ with increments 0.1, we obtain 24 bins to which we assign numeric symbols going from 1 to 24. Then, we associate one symbol to each log-return, depending on the bin it belongs to. As seen in Figure 9, second table, most of the symbols orbit around the intervals closest to zero, since most of the variations of the time series are relatively small.

In order to calculate the simplest probabilities, $p(i_n)$ appearing in (9), we just need to count how many times each symbol appears in vector X_n and then divide by the total number of occurrences. As an example, from the first 10 lines of data shown in Figure 9, the symbol 15 appears 4 times. In order to calculate $p\left(i_{n+1},i_n\right)$,

we must count how many times a particular combination of symbols, (a, b), appears in the joint columns X_{n+1} and X_n. As an example, in the first ten lines of such columns, the combination $(15, 15)$ appears zero times, the combination $(15, 16)$ appears 4 times, the combination $(16, 15)$ appears 4 times, and the combination $(16, 16)$ appears two times.

Date	X_{n+1}	X_n	Y_n
04/01/2007	0.0025	−0.0048	0.0044
05/01/2007	−0.0083	0.0025	0.0001
08/01/2007	0.0033	−0.0083	−0.0127
09/01/2007	−0.0042	0.0033	−0.0053
10/01/2007	0.0073	−0.0042	0.0056
11/01/2007	0.0044	0.0073	−0.0106
12/01/2007	−0.0066	0.0044	0.0177
16/01/2007	0.0083	−0.0066	0.0137
17/01/2007	0.0008	0.0083	−0.0012
18/01/2007	−0.0058	0.0008	−0.0048
⋮	⋮	⋮	⋮

$\longrightarrow$

X_{n+1}	X_n	Y_n
16	15	16
15	16	16
16	15	15
15	16	15
16	15	16
16	16	15
15	16	16
16	15	16
16	16	15
15	16	15
⋮	⋮	⋮

Fig. 9. Table on the right: first log-returns of the time series of the J.P. Morgan (X_{n+1}), of its lagged values by one day (X_n), and of the log-returns of the Deutsche Bank (Y_n) lagged by one day. Table on the right: symbols are associated to each value of the log-return, inside an interval $[-1.5, 0.8]$ with increments 0.1.

For the whole data, we have the following probabilities and joint probabilities shown in Figure 10. Here, it becomes clearer why, sometimes, it is best to use a binning of larger size in order to calculate Transfer Entropy, since when one has too many binnings, the chance of having particular combinations drop very quickly, making the calculation of probabilities less informing.

X_n	$Freq$	$p(i_n)$
13	1	0.0007
14	13	0.0086
15	757	0.5020
16	720	0.4775
17	14	0.0093
18	3	0.0020

X_{n+1}	X_n	$Freq$	$p(i_{n+1}, i_n)$
13	15	1	0.0007
14	14	1	0.0007
14	15	7	0.0046
14	16	3	0.0020
14	17	2	0.0013
15	14	5	0.0033
15	15	338	0.2241
15	16	408	0.2706
15	17	5	0.0033
15	18	1	0.0007
16	14	5	0.0033
16	15	404	0.2679
16	16	304	0.2016
16	17	5	0.0033
16	18	2	0.0013
17	14	2	0.0013
17	15	5	0.0033
17	16	5	0.0033
17	17	2	0.0013
18	13	1	0.0007
18	15	2	0.0013

X_n	Y_n	$Freq$	$p(i_n, j_n)$
13	14	1	0.0007
14	14	2	0.0013
14	15	11	0.0073
15	14	10	0.0066
15	15	473	0.3137
15	16	271	0.1797
15	17	3	0.0020
16	15	289	0.1916
16	16	421	0.2792
16	17	10	0.0066
17	14	2	0.0013
17	15	4	0.0027
17	16	6	0.0040
17	17	1	0.0007
17	18	1	0.0007
18	16	2	0.0013
18	17	1	0.0007

X_{n+1}	X_n	Y_n	$Freq$	$p(i_{n+1}, i_n, j_n)$
13	15	15	1	0.0007
14	14	15	1	0.0007
14	15	14	1	0.0007
14	15	15	3	0.0020
14	15	16	3	0.0020
14	16	15	1	0.0007
14	16	16	1	0.0007
14	16	17	1	0.0007
14	17	15	1	0.0007
14	17	17	1	0.0007
15	14	14	1	0.0007
15	14	15	4	0.0027
15	15	14	5	0.0033
15	15	15	216	0.1432
15	15	16	115	0.0763
15	15	17	2	0.0013
15	16	15	154	0.1021
15	16	16	247	0.1638
15	16	17	7	0.0046
15	17	14	1	0.0007
15	17	15	1	0.0007
15	17	16	3	0.0020
15	18	16	1	0.0007
16	14	14	1	0.0007

X_{n+1}	X_n	Y_n	$Freq$	$p(i_{n+1}, i_n, j_n)$
16	14	15	4	0.0027
16	15	14	3	0.0020
16	15	15	249	0.1651
16	15	16	151	0.1001
16	15	17	1	0.0007
16	16	15	132	0.0875
16	16	16	170	0.1127
16	16	17	2	0.0013
16	17	14	1	0.0007
16	17	15	1	0.0007
16	17	16	2	0.0013
16	17	18	1	0.0007
16	18	16	1	0.0007
16	18	17	1	0.0007
17	14	15	2	0.0013
17	15	14	1	0.0007
17	15	15	3	0.0020
17	15	16	1	0.0007
17	16	15	2	0.0013
17	16	16	3	0.0020
17	17	15	1	0.0007
17	17	16	1	0.0007
18	13	14	1	0.0007
18	15	15	1	0.0007
18	15	16	1	0.0007

Fig. 10. Probabilities and joint probabilities of the times series X_{n+1}, X_n, and Y_n.

We now sum over all combinations of the components of X_{n+1}, X_n, and Y_n using definition (9), obtaining as a result $TE_{177\to 4} = 0.0155$. This result indicates the average amount of information transferred from the Deustche Bank to the J.P. Morgan which was not already contained in the information of the past state of the J.P. Morgan one day before. Doing the same for all possible combinations of stocks, one obtains a Transfer Entropy matrix, which is represented in terms of false colors in Figure 11a.

Here, like in the calculation of the Shannon Entropy, the size of the bins used in the calculations of the probabilities changes the resulting Transfer Entropy (TE). The calculations we have shown in figures 9 and 10 are relative to a choice of binning of size 0.1. In order to compare the resulting TE matrix with that of another choice for binning, we calculated the TE for binning size 0.02, what leads to a much larger number of bins and to a much longer calculation time. The resulting TE matrix for binning 0.02 is plotted in Figure 11b. The two TE matrices are not very different, with the main dissimilarities being due to scale. The visualization for binning size 0.1 is sharper than the one obtained using binning size 0.02. In what follows, we shall consider

binning size 0.1 throughout the calculations, since it demands less computation time and delivers clearer results in comparison with the ones obtained for some smaller sized binnings.

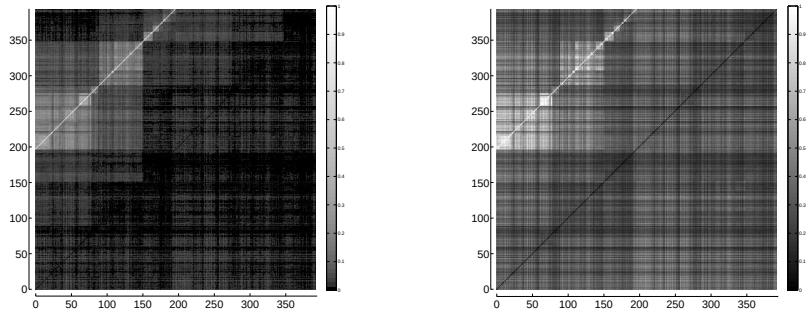

Figure 11a Figure 11b

Fig. 11. False color representations of the Transfer Entropy (TE) matrix. In Figure 11a, we have the representation of the TE for a binning of size 0.1; in Figure 11b, we have the representation of the TE for a binning of size 0.02.

4.3 Effective Transfer Entropy

Transfer Entropy matrices usually contain much noise, due to the finite size of data used in their calculation, non-stationarity of data, and other possible effects, and we must also consider that stocks that have more entropy, what is associated with higher volatility, naturally transfer more entropy to the others. We may eliminate some of these effects if we calculate the Transfer Entropy of randomized time series, where the elements of each time series are randomly shuffled so as to break any causality relation between variables but maintain the individual probability distributions of each time series. The original Transfer Entropy matrix is represented in Figure 12a. The result of the average of 25 simulations with randomized data appears in Figure 12b. We only calculated 25 simulations because the calculations are very computationally demanding, and because the results for each simulation are very similar. Then, an Effective Transfer Entropy matrix (ETE) may be calculated by subtracting the Randomized Transfer Entropy matrix (RTE) from the Transfer Entropy matrix (TE):

$$ETE_{Y\to X} = TE_{Y\to X} - RTE_{Y\to X} \ . \tag{10}$$

The result is shown in Figure 12c.

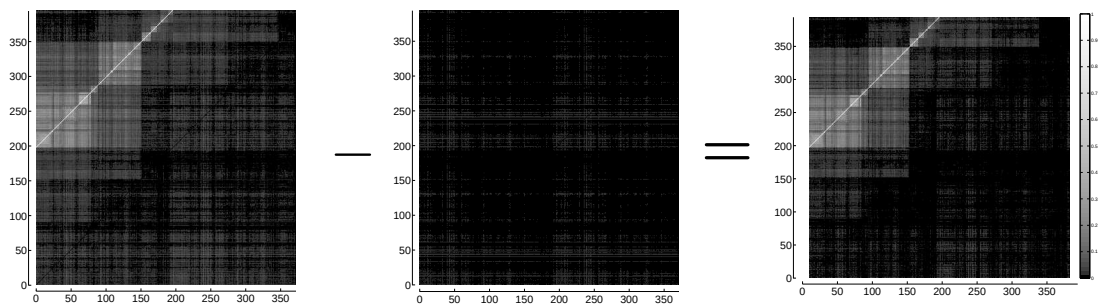

Figure 12a Figure 12b Figure 12c

Fig. 12. False color representations of the Transfer Entropy matrix (Figure 12a), of the Randomized Transfer Entropy Matrix (Figure 12b, the average of 25 simulations with randomized data), and of the Effective Transfer Entropy (Figure 12c).

The main feature of the representation of the Effective Transfer Entropy matrix (or of the Transfer Entropy matrix) is that it is clearly not symmetric. The second one is that the highest results are all in the quadrant on the left topmost corner (Quadrant 12). That is the quadrant related with the Effective Transfer Entropy (ETE) from the lagged stocks to the original ones. The main diagonal expresses the ETE from one stock to

itself on the next day, which, by the very construction of the measure being used, is expected to be high. But Quadrant 12 also shows that there are larger transfers of entropy from lagged stocks to the other ones than between stocks on the same day. We must remind ourselves that we are dealing here with the daily closing prices of stocks, and that the interaction of prices of stocks, and their reactions to news, usually occur at high frequency. Here, we watch the effects that a whole day of negotiation of a stock has on the others. Figure 13a shows a closer look at the ETE of the stocks on stocks on the same day, what corresponds to the quadrant on the bottom left (Quadrant 11), and from lagged to original stocks, in Figure 13b (Quadrant 12).

Analyzing Quadrant 12 (Figure 13b), we may see again the structures due to geographical positions, with clusters related with stocks from the USA (1 to 79), Canada (80 to 89), Europe (91 to 152), Japan (153 to 165), Hong Kong (166 to 174), Singapore (177 to 179), and Australia (180 to 197). We also detect some ETE from lagged stocks from the USA to stocks from Canada and Europe, from lagged stocks from Europe to stocks from the USA and Canada and, with a smaller strength, from lagged stocks from Europe to stocks from Australasia, and transfer of entropy within the Australasian stocks. Quadrant 11 (Figure 13a) shows much smaller values, but one can see a clear influence of Japan (153-165) on North America (1-89) and Europe (91-152), and also some influence from Europe to the USA. A very light influence may be seen from the USA to itself on the next day, Canada, and Europe, but it is already hard to distinguish this influence from noise. There are negative values of ETE, what means that the Transfer Entropy calculated is smaller than what would be expected from noise.

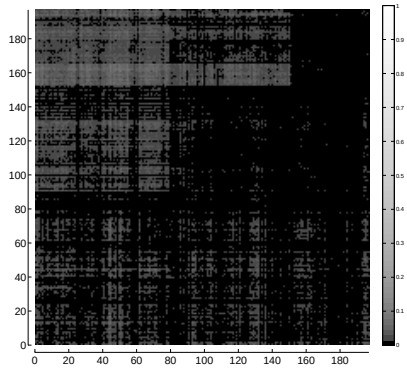

Figure 13a

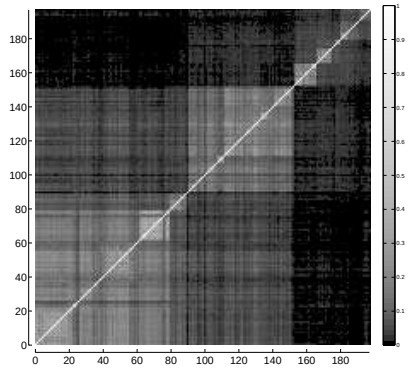

Figure 13b

Fig. 13. False color representations of two quadrants of the Transfer Entropy matrix. Figure 13a shows the quadrant of the ETEs from stocks to the stocks at the same day (Quadrant 11), and Figure 13b shows the quadrant of ETEs from lagged stocks to original ones (Quadrant 12).

There are intra-sector structures inside each block, but this may be best analyzed by using thresholds above which we assign value 1 to ETEs, and bellow which we assign value 0. Figure 14 shows the resulting false color maps for thresholds 0.3, 0.2, and 0.1. The structures previously described are all quite clear in these graphs for Quadrant 12. We shall discuss in more detail the structure that appears from threshold 0.4, which is not shown in Figure 14, since it has very few connections, with the main ones being the ETEs from lagged stocks to their original counterparts. At this threshold, for stocks of the USA, there is already an ETE from the State Street (Fiduciary Bank) to the Fifth Third Bancorp (Super-regional Bank), and mutual exchanges of ETE between Prudential Financial (Life/Health Insurance) and MetLife (Multi-line Insurance), between Itau Unibanco Holding and Banco Bradesco (both stocks of two Brazilian banks negotiated in the NYSE), and between HCP and Ventas (both REITS-Health Care). There is also a dense cluster of REITS, with ETEs flowing from one to the other, but not from all of them to all of them, composed of Apartment Investment & Management and Equity Residential (REITS-Apartments), Boston Properties (REITS-Office Property), Simon Property Group (REITS-Regional Malls), Kimco Realty (REITS-Shopping Centers), and Vornado Realty Trust (REITS-Diversified). The most pointed to stock is the one of Vornado Realty Trust. From Spain, we have a mutual relation between Banco Bilbao Vizcaya Argentaria and Banco Santander (both large Commercial Banks). From Japan, there is a pair of interdependent stocks, Mitsubishi UFJ Financial (Diversified Banking Institution) and Sumitomo Mitsui Financial (Commercial Banks), and a trio, consisting of Mitsui Fudosan and

Sumitomo Realty & Development (both Real Estate Operation/Development), and Mitsubishi Estate (Real Estate Management/Services). A last pair occurs for Hong Kong, between the stocks of China Construction Bank and Industrial & Commercial Bank of China (both Commercial Banks).

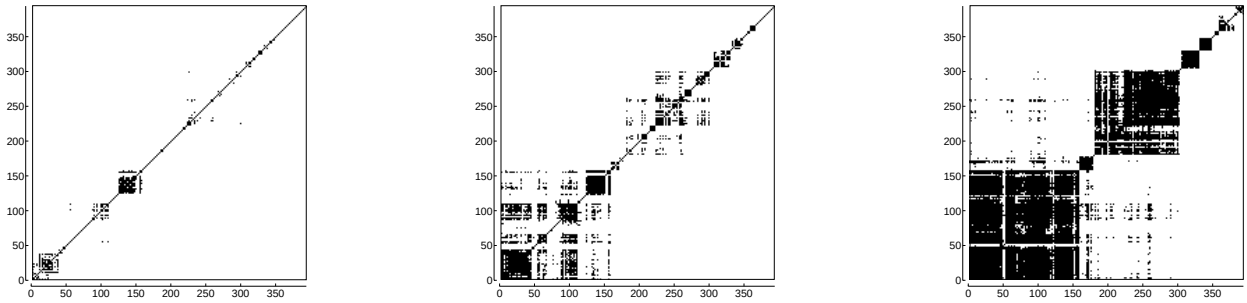

Fig. 14. Elements of Quadrant 12 (lagged-original sector) of the ETE matrix that are above some threshold. Points above the threshold are represent in white, and points bellow it are represented in black. The figures correspond to the following thresholds: 0.3, 0.2, and 0.1, respectively.

4.4 Normalized Transfer Entropy and Asset Graphs

We may again try to produce a map of the nodes according to distances between stocks. The problem now is that distance is a symmetric measure, and the Effective Transfer Entropy is not. Another problem is that the ETE is not normalized. We may correct the latter problem by defining the *Normalized Transfer Entropy*, which uses another measure derived from the Shannon entropy, called *Conditional Entropy*, which is defined in the following way: the Conditional Entropy of X given Y is the average uncertainty in the outcome of a measurement x of X when the measure y of Y is known:

$$H_{X|Y} = -\sum_{i_n,j_n} p\left(i_n, j_n\right) \log_2 p\left(i_n|j_n\right) = -\sum_{i_n,j_n} p\left(i_n, j_n\right) \log_2 \frac{p\left(i_n, j_n\right)}{p\left(j_n\right)} \ . \tag{11}$$

Based on this concept, we may define the *Normalized Transfer Entropy* as

$$NTE = \frac{ETE_{Y\to X}}{H_{X^F|X^P}} \ , \tag{12}$$

where $H_{X^F|X^P}$ is the conditional entropy of the future of X on its past, what we may write as

$$H_{X^F|X^P} = -\sum_{i_n,j_n} p\left(i_{n+1}, i_n\right) \log_2 \frac{p\left(i_{n+1}, i_n\right)}{p\left(i_n\right)} \ . \tag{13}$$

The resulting values are always between -1 and 1. Using now definition (2), we may define elements d_{ij}. However, the resulting matrix does not necessarily have $d_{ii} = 0$, what is a necessary condition for it to be a distance measure. So we must fix that by setting all diagonal elements to zero. The resulting matrix is still not symmetric, and we symmetrize the matrix by setting $d_{ij} = d_{ji}$ if $d_{ij} > d_{ji}$ and $d_{ji} = d_{ij}$, otherwise, what means that we always consider the smallest between the two values d_{ij} and d_{ji} to be the distance between i and j. The resulting distance matrix is then used, applying (3), in order to calculate a set of coordinates for each stock as a node in a space where distances are similar to the ones given by the symmetrized distance matrix.

Figure 15 shows the stocks (original and lagged ones) plotted in two-dimensional graphs. In Figure 15a, original stocks are in white and their lagged values are in black. As expected from the results we saw for the ETE matrix, lagged and original values are very close one to the other. This is in strong contrast with the results obtained using correlation (Figure 3) where original and lagged stocks occupy very distinct positions. In Figure 15b, the lagged stocks were removed, and continents are highlighted with different shades of gray: white for America, black for Europe and gray for Australasia. Another difference that may be seen here is that Australasia seems closer to America than Europe.

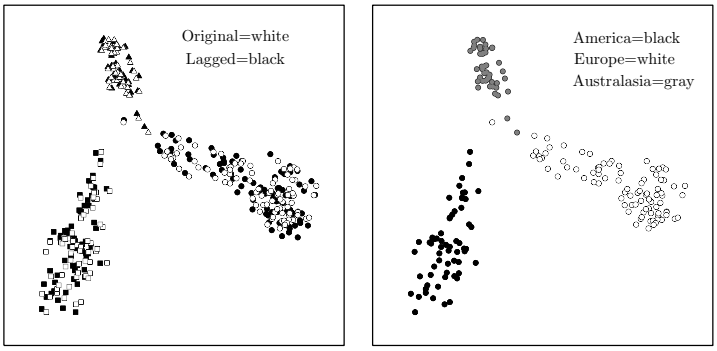

Figure 15a Figure 15b

Fig. 15. Two dimensional representation of the stocks as nodes in coordinates that simulate the distances between them obtained from the ETE. In Figure 15a, white dots (America), squares (Europe), and triangles (Australasia) represent the original log-returns, and black dots, squares, and triangles represent their lagged values by one day. In Figure 15b, continents are represented: stocks belonging to America are represented as white dots, stocks belonging to Europe are represented as black dots, and stocks belonging to Eurasia are represented by gray dots. The lagged data were removed in the second graph.

Once more, by using thresholds, we are able to filter some of the information in such graph, and we may also build asset graphs with connections between some nodes. Here, we choose the values of the ETE and not of the distance matrix in order to establish thresholds. The first reason is because the distance matrix highly modifies the original relations between stocks and lagged stocks, and the second one is that the distance values do not vary very linearly. For a choice of thresholds 0.4, 0.3, and 0.2 for the ETE, deleting all edges below these values and all unconnected nodes after that removal, we obtain the graphs in Figure 16. The number of connections (edges) increases dramatically for higher values of the threshold, approaching a limit at which all nodes are connected.

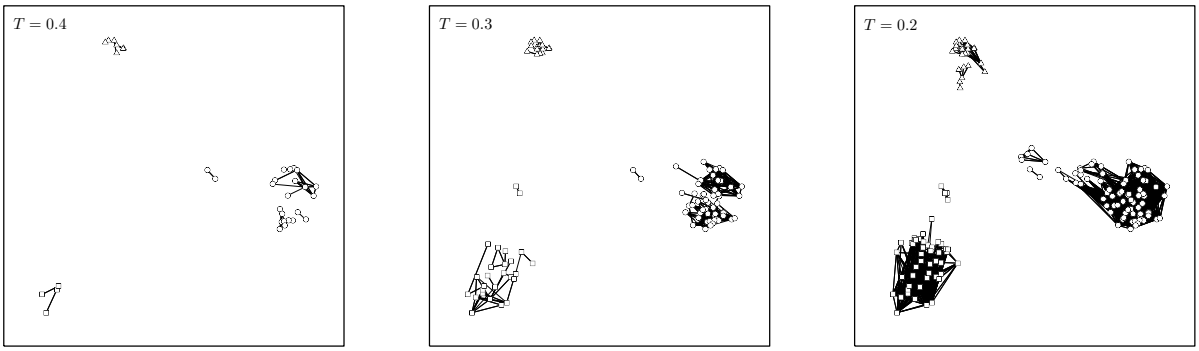

Fig. 16. Two dimensional representation of the asset graphs of the ETEs between stocks for thresholds 0.4, 0.3, and 0.2.

In Figure 17, we take a closer look at the relationships between the stocks at threshold 0.4. At the lower right corner, there are three small clusters of stocks from the USA in the same rectangle. The first one is the transfer entropy between stocks of Well Fargo (Super-Regional Bank) to the stocks of J.P. Morgan Chase (Diversified Banking Institution); the second one is a cluster of Insurance companies (Hartford, Principal, Met Life, Prudential, and Lincoln); the third one is a small cluster of Super-Regional Banks (Huntington Bancshares, Fifth Third, and Sun Trust). At the top right rectangle, there are two clusters of stocks from the USA. The first one is a large cluster of REITS (Real Estate Investment Trusts), comprising Avalon Bay, Equity Residential, Apartment Investment & Management, Kimko Realty, Macerich, Simon Property Group, Boston Properties, Prologis, and Vornado Realty Trust; the second one is a pair of two REITS of Health Care: HCP and Ventas. At the center of the graph, we have a rectangle with the pair Banco Bradesco and Itau Unibanco, which are the stocks of major Commercial Banks based in Brazil negotiated in the New York Stock Exchange. At the lower left of the graph, there are two pairs: one of Diversified Banking Institutions from

France (Societé Générale and BNP Paribas) and one of major Commercial Banks from Spain (Banco Bilbao Vizcaya Argentaria and Banco Santander). At the top left, we have the last clusters; the first one, a pair of stocks from Japan: Mitsubishi UFJ Financial Group (Diversified Banking Institution) and of Sumimoto Mitsui Financial Group (Commercial Bank); the second one, whose elements are Mitsubishi Estate, Mitsui Fudosan, and Sumitomo Realty & Development, is a cluster of Real Estate operations, management and services firms; the third one is a pair of two Commercial Banks from Hong Kong: Industrial & Commercial Bank of China and China Construction Bank. It is to be noticed that most relations are reciprocate, although the ETE between stocks is rarely very similar.

We shall not make a deeper analysis of the remaining asset graphs, but one can see that integration begins inside countries, with the exception of certain countries from Europe, and then goes continental. Only at threshold 0.1 and below, we start having intercontinental integration. This may be due to differences in operation hours of the stock exchanges, to geographical, economic and cultural relations, or to other factors we failed to contemplate (see, for instance, Sandoval, 2012a for a discussion).

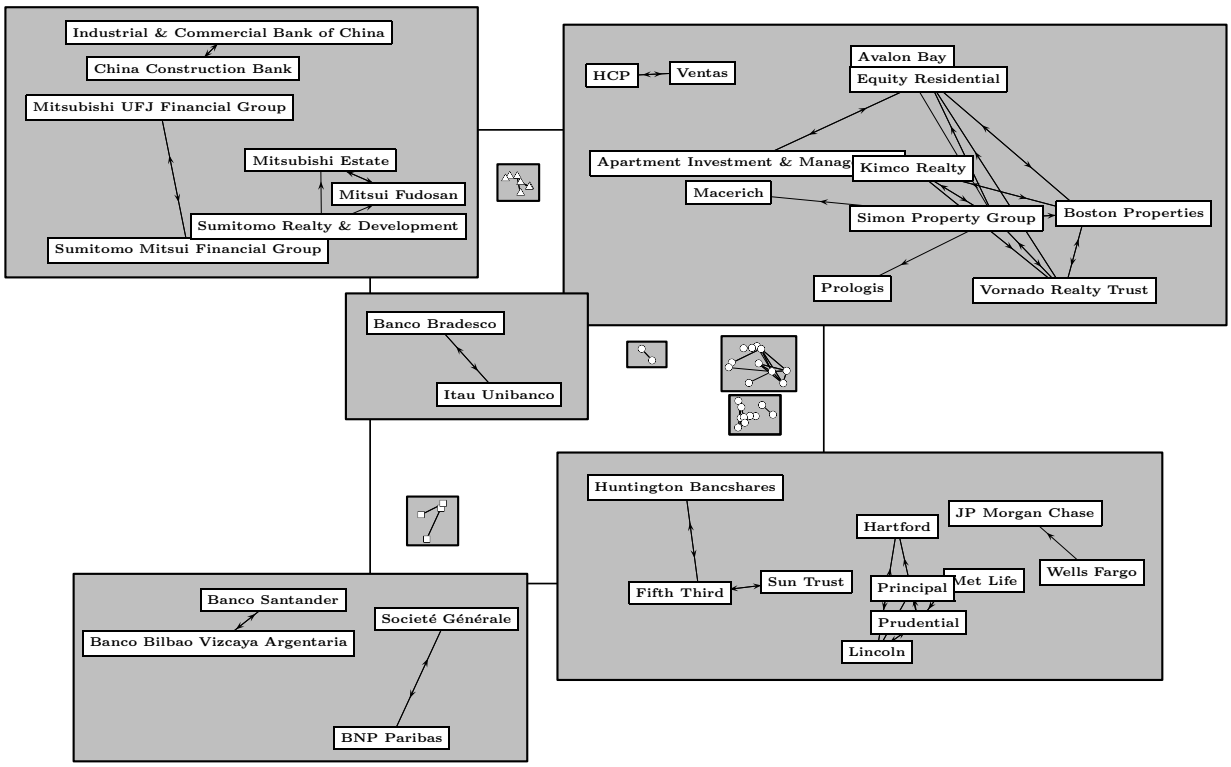

Fig. 17. Detailed look at the ETEs between stocks at threshold 0.4. Each group of stocks is located in a magnified window, with the names of each stock close to the position it occupies in the complete network.

4.5 Centralities

The measures of centrality presented in Section 3 are appropriate for an undirected network, like the one obtained by using correlation, but the networks built using Effective Transfer Entropy are directed nodes, that have either ingoing edges to a node, outgoing edges from the node, or both. So, centrality measures often break down into ingoing and outgoing ones. As an example, a node may be highly central with respect to pointing at other nodes, like the Google search page; these are called *hubs*. Other nodes may have many other nodes pointing at it, as in the case of a highly cited article in a network of citations; these are called *authorities*. Each one is central in a different way, and a node may be central according to both criteria. Node degree, for example, may be broken in two measures: In Node Degree (ND_{in}), which measures the sum of all ingoing edges to a certain node, and Out Node Degree (ND_{out}), which measures the sum of all outgoing edges from a node. In a similar way, one defines In Eigenvector Centrality (EC_{in}) and Out Eigenvector Centrality (EC_{in}), and In Harmonic Closeness (HC_{in}) and Out Harmonic Closeness (HC_{in}). Betweenness Centrality is now calculated

along directed paths only, and it is called Directed Betweenness Centrality, (BC_{dir}).

As we said before, when applying centrality measures to asset graphs, those measures vary according to the chosen value for the threshold. As extreme examples, if the threshold is such that the network has very few nodes, Node Centrality, for example, will also be low. If the threshold value is such that every node is connected to every other node, then all Node Degrees will be the same: the number of all connections made between the nodes. It has been shown empirically (Sandoval, 2013) that one gets the most information about a set of nodes if one considers asset graphs whose thresholds are close to the minimum or the maximum of the values obtained through simulations with randomized data. We may rephrase it by saying that we obtain more information of a network when we consider its limit to results obtained from noise. From the simulations we have made in order to calculate the Effective Transfer Entropy, we could check that the largest values of Transfer Entropy for randomized data are close to 0.05 for the choice of bins with size 0.1 (Figure 12a). So, we shall consider here the centrality measures that were mentioned applied to the directed networks obtained from the Effective Transfer Entropy with threshold 0.05. The results are plotted in Figure 18. As the values of different centralities may vary a lot (from 3 to 153 for ND_{in} and from 0 to 1317 for BC_{dir}), we normalize all centrality measures by setting their maxima to one. For all but Directed Betweenness Centrality, stocks belonging to the Americas and to Europe appear more central.

Table 2 presents the most central stocks according to each centrality measure. Only the first five stocks are shown (more, in case of draws). Lagged stocks appear with an $*$ besides the names of the companies. Since we are considering only the strong values of transfer entropy, and since asset graphs do not involve the nodes that are not connected, this excludes all connections, except the ones between lagged and original log-returns. So, all in degrees are of original stocks and all out degrees (including Directed Betweenness) are of lagged stocks. For out degrees, insurance companies occupy the top positions, together with some banks, all of them belonging to European or to U.S. companies. For in degrees, we see a predominance of banks, but insurance companies also occupy top positions. This means there is a tendency of entropy being transferred from insurance companies to banks. For Directed Betweeenness, the top positions are occupied by major European banks and also by other types of companies.

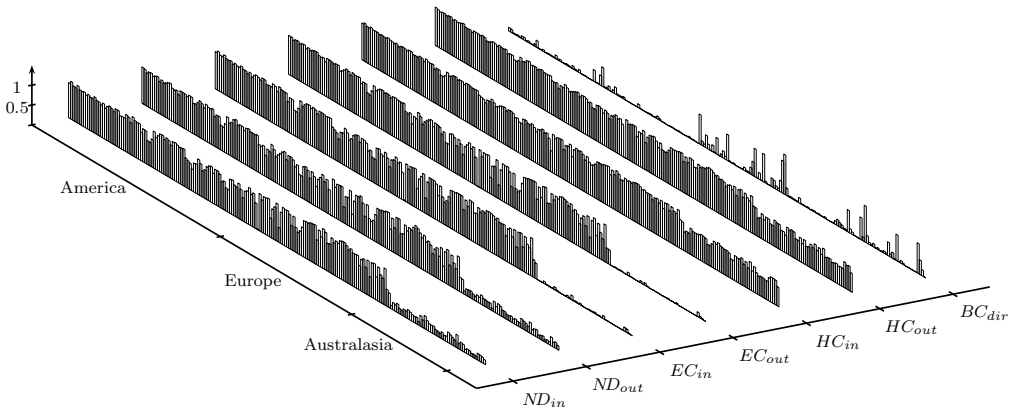

Fig. 18. Centrality measures of stocks for the asset graph with threshold 0.05. All measures were normalized so as to have maximum one.

Centrality	Company	Country	Industry	Sub-Industry
In Node Degree				
153	Credit Suisse Group AG	Switzerland	Banks	Diversified Banking Inst
150	Deutsche Bank AG	Germany	Banks	Diversified Banking Inst
149	Invesco	USA	Diversified Finan Serv	Invest Mgmnt/Advis Serv
149	ING Groep NV	Netherlands	Insurance	Life/Health Insurance
149	KBC Groep NV	Belgium	Banks	Commer Banks Non-US
Out Node Degree				
160	ING Groep NV*	Netherlands	Insurance	Life/Health Insurance
158	Hartford Financial Services Group*	USA	Insurance	Multi-line Insurance
154	KBC Groep*	Belgium	Banks	Commer Banks Non-US
152	Genworth Financial*	USA	Insurance	Multi-line Insurance
151	Lincoln National Corp*	USA	Insurance	Life/Health Insurance
In Eigenvector				
11.99	Invesco	USA	Diversified Finan Serv	Invest Mgmnt/Advis Serv
11.91	Credit Suisse Group AG	Switzerland	Banks	Diversified Banking Inst
11.86	Hartford Financial Services Group	USA	Insurance	Multi-line Insurance
11.85	Lincoln National Corp	USA	Insurance	Life/Health Insurance
11.83	MetLife	USA	Insurance	Multi-line Insurance
Out Eigenvector				
0.094	Hartford Financial Services Group*	USA	Insurance	Multi-line Insurance
0.094	Lincoln National Corp*	USA	Insurance	Life/Health Insurance
0.093	Invesco*	USA	Diversified Finan Serv	Invest Mgmnt/Advis Serv
0.093	MetLife*	USA	Insurance	Multi-line Insurance
0.093	ING Groep*	Netherlands	Insurance	Life/Health Insurance
0.093	Genworth Financial*	USA	Insurance	Multi-line Insurance
0.093	Principal Financial Group*	USA	Insurance	Life/Health Insurance
0.093	UBS*	Switzerland	Banks	Diversified Banking Inst
0.093	Prudential Financial*	USA	Insurance	Life/Health Insurance
0.093	Ameriprise Financial*	USA	Diversified Finan Serv	Invest Mgmnt/Advis Serv
In Harmonic Closeness				
174.00	Credit Suisse Group AG	Switzerland	Banks	Diversified Banking Inst
172.5	Deutsche Bank AG	Germany	Banks	Diversified Banking Inst
171.8	KBC Groep NV	Belgium	Banks	Commer Banks Non-US
171.2	ING Groep NV	Netherlands	Insurance	Life/Health Insurance
170.5	Commerzbank AG	Germany	Banks	Commer Banks Non-US
Out Harmonic Closeness				
178	ING Groep*	Netherlands	Insurance	Life/Health Insurance
177	Hartford Financial Services Group*	USA	Insurance	Multi-line Insurance
175	KBC Groep*	Belgium	Banks	Commer Banks Non-US
174	Genworth Financial*	USA	Insurance	Multi-line Insurance
173	Barclays*	UK	Banks	Diversified Banking Inst
Directed Betweenness				
1317	KBC Groep*	Belgium	Banks	Commer Banks Non-US
1202	China Construction Bank Corp*	Hong Kong	Banks	Commer Banks Non-US
1074	ING Groep*	Netherlands	Insurance	Life/Health Insurance
998	Goodman Group*	Australia	REITS	REITS-Diversified
984	Barclays*	UK	Banks	Diversified Banking Inst

Table 2. Classification of stocks with highest centrality measures, the countries they belong to, their industry and sub-industry classifications, for asset graphs based on threshold 0.05. Only the five stocks with highest centrality values are shown (more, in case of draws).

Figure 19 shows the normalized values of the centrality measures for the asset graph obtained with threshold 0.1. The figure has a smaller number of stocks, since there are slightly fewer nodes for this value of the threshold. One may notice a sharp drop in values for Eigenvector Centralities in this asset graph. Table 3 shows the most central stocks according to each centrality measure for this choice of binning. Only the first five stocks are shown (more, in case of draws). In all centrality measures, insurance companies occupy the first positions, and the same stocks usually occupy these positions, except for Provident Financial.

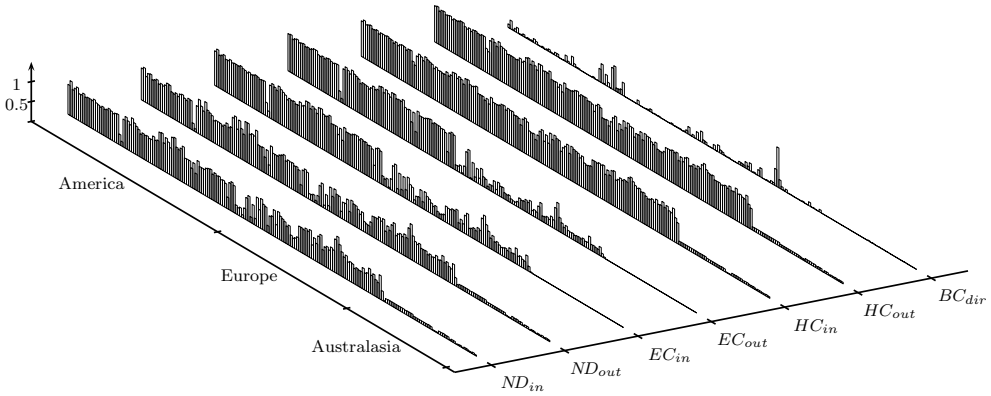

Fig. 19. Centrality measures of stocks for the asset graph with threshold 0.1. All measures were normalized so as to have maximum one.

Centrality	Company	Country	Industry	Sub-Industry
		In Node Degree		
113	Lincoln National Corp	USA	Insurance	Life/Health Insurance
110	Provident Financial	UK	Diversified Finan Serv	Finance-Consumer Loans
106	ING Groep NV	Netherlands	Insurance	Life/Health Insurance
105	Hartford Financial Services Group	USA	Insurance	Multi-line Insurance
103	Genworth Financial	USA	Insurance	Multi-line Insurance
		Out Node Degree		
120	Provident Financial*	UK	Diversified Finan Serv*	Finance-Consumer Loans*
120	Lincoln National Corp*	USA	Insurance	Life/Health Insurance*
118	Hartford Financial Services Group*	USA	Insurance	Multi-line Insurance
113	MetLife*	USA	Insurance	Multi-line Insurance
113	Prudential Financial*	USA	Insurance	Life/Health Insurance
		In Eigenvector		
9.91	ING Groep NV	Netherlands	Insurance	Life/Health Insurance
9.81	Lincoln National Corp	USA	Insurance	Life/Health Insurance
9.58	Provident Financial	UK	Diversified Finan Serv	Finance-Consumer Loans
9.57	Hartford Financial Services Group	USA	Insurance	Multi-line Insurance
9.44	Aegon NV	Netherlands	Insurance	Multi-line Insurance
		Out Eigenvector		
0.126	Provident Financial*	UK	Diversified Finan Serv	Finance-Consumer Loans
0.126	Lincoln National Corp*	USA	Insurance	Life/Health Insurance
0.125	Hartford Financial Services Group*	USA	Insurance	Multi-line Insurance
0.124	MetLife*	USA	Insurance	Multi-line Insurance
0.124	Prudential Financial*	USA	Insurance	Life/Health Insurance
		In Harmonic Closeness		
131.0	Provident Financial	UK	Diversified Finan Serv	Finance-Consumer Loans
129.5	Lincoln National Corp	USA	Insurance	Life/Health Insurance
127.5	Hartford Financial Services Group	USA	Insurance	Multi-line Insurance
127.0	MetLife	USA	Insurance	Multi-line Insurance
126.0	Prudential Financial	USA	Insurance	Life/Health Insurance
		Out Harmonic Closeness		
134.5	Lincoln National Corp*	USA	Insurance	Life/Health Insurance
134.5	Provident Financial*	UK	Diversified Finan Serv	Finance-Consumer Loans
133.5	Genworth Financial*	USA	Insurance	Multi-line Insurance
131	Hartford Financial Services Group*	USA	Insurance	Multi-line Insurance
131	Prudential Financial*	USA	Insurance	Life/Health Insurance
		Directed Betweenness		
1486	ING Groep*	Netherlands	Insurance	Life/Health Insurance
911	Lincoln National Corp*	USA	Insurance	Life/Health Insurance
802	Provident Financial*	USA	Diversified Fin. Serv.	Finance - Consumer Loans
705	Hartford Financial Services Group*	USA	Insurance	Multi-line Insurance
636	Aegon*	Netherlands	Insurance	Multi-line Insurance

Table 3. Classification of stocks with highest centrality measures, the countries they belong to, their industry and sub-industry classifications, for asset graphs based on threshold 0.1. Only the five stocks with highest centrality values are shown (more, in case of draws).

For threshold 0.2, there is also a preponderance on insurance companies and banks from the USA, and for thresholds 0.3 and 0.4, there are mostly banks and REITS occupying the first positions, also due to the fact that they are some of the only nodes that are part of the asset graphs at these threshold values.

The centrality measures we have considered thus far in this section do not take into account the strength of the connections between the nodes. There are centrality measures that take that into account, being the main one called *Node Strength* (NS), which, in undirected networks, is the sum of all connections made by a node. For directed networks, we have the *In Node Strength* (NS_{in}), which measures the sum of all ingoing connections to a node, and the *Out Node Strength* (NS_{out}), which measures the sum of all outgoing connections from a node. These are centrality measures that can be applied to the whole network, including all nodes. Figure 20 shows the results for both centrality measures, and Table 4 shows the top five stocks according to each node centrality. We used ETE in the calculations. Had we used TE instead, the results would be the same.

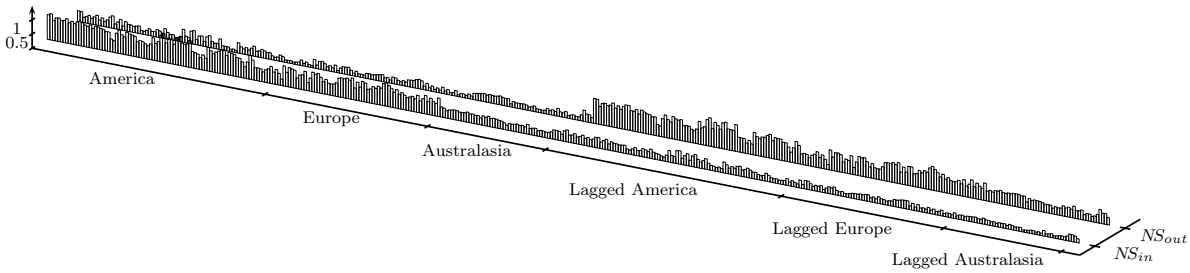

Fig. 20. Node Strengths (in and out) for the whole network. Both measures were normalized so as to have maximum one.

Centrality	Company	Country	Industry	Sub-Industry
	In Node Strength			
30.34	Hartford Financial Services Group	USA	Insurance	Multi-line Insurance
29.86	Lincoln National Corp	USA	Insurance	Life/Health Insurance
29.77	Prudential Financial	USA	Insurance	Life/Health Insurance
29.22	Principal Financial Group	USA	Insurance	Life/Health Insurance
27.87	Citigroup	USA	Banks	Diversified Banking Inst
	Out Node Strength			
30.16	Hartford Financial Services Group *	USA	Insurance	Multi-line Insurance
28.71	Prudential Financial *	USA	Insurance	Life/Health Insurance
27.83	Lincoln National *	USA	Insurance	Life/Health Insurance
27.31	Principal Financial Group *	USA	Insurance	Life/Health Insurance
26.57	ING Groep NV *	Netherlands	Insurance	Life/Health Insurance

Table 4. Top five stocks according to In Node Strength and to Out Node Strength, the countries they belong to, their industry and sub-industry classifications. Nodes related with lagged stocks have an asterisk beside their names. Calculations were based on the ETEs between stocks.

The five top stocks for In Node Strength are those of Insurance Companies, qualified as authorities, which are nodes to which many other nodes point, and with high values of ETE, what means that there is a large amount of information flowing into the log-returns of those stocks. For Out Node Strength, again insurance companies dominate, what means that they send much information into the prices of the other stocks (they are also hubs).

5 Relations with economies in crisis

Economic broadcasts of the past few years constantly warned of the dangers of a new global financial crisis that may be triggered by the failure of some European countries to pay their sovereign debts. It is not completely clear how far reaching a default by one of those countries could be, and which institutions are more vulnerable to that. Using networks based on financial loans and debts between banks, researchers can try to gauge some of the consequences of defaults in banks, but, as said in the introduction, networks built on loans and debts do not account for a myriad of other economical facts that define the relationships between financial institutions. So, in order to attempt to study those relations, we shall build networks based on the ETEs between the 197 major financial institutions considered until now together with all financial institutions listed in Bloomberg of some of those countries in crisis, after a liquidity filter. The aim is to investigate which of the main financial institutions receive more entropy from the financial institutions of those countries, meaning that the prices of

stocks from those target institutions are much influenced by the prices of institutions that might be in danger of collapse. Of course, we are not saying here that the institutions being considered that belong to one of the countries in crisis might default; we just analyze what could happen if they did.

The countries we shall consider here are Greece, Cyprus, Ireland, Spain, Portugal, and Italy. We will do a separate analysis for each country, following the same procedures. First, we remove the stocks belonging to the country in crisis from the original network of financial institutions; then we add to this network all stocks that belong to the country in crisis and that are listed in Bloomberg. The number os stocks from each country are restrained by the data available and by the liquidity of those stocks. The second condition eliminates many of the time series available, particularly in less developed stock markets.

Greece is represented by 17 stocks, including the Bank of Greece, which is removed from the 197 original stocks of financial companies. For Cyprus, we obtain the time series of 20 stocks, after removing the less liquid ones. Spain is one of the main players in the international fears for the world economic market; we remove the stocks belonging to Spanish companies (four of them) from the bulk of main stocks and then add 26 stocks of financial companies from that country, including the ones that have been previously removed. Portugal is also an important country in the monitoring for an economic crisis since its institutions have deep connections with Spanish companies. In order to study the influence of its stocks on other stocks of main financial companies, we first remove the one stock belonging to Portugal in that group, that of the Banco Espírito Santo. Then we add to the data the log-returns of five major Portuguese banks, including the one that had been removed from the main block. The country in this group with the largest number of companies that take part of the original data set, 6 of them, is Italy, for which we start by removing those stocks from the main block, including the 6 original ones. Then we add 61 stocks belonging to the financial sector which are negotiated in Italy and which survive the liquidity filter. For Ireland, we have four stocks that survive the liquidity filter.

The Transfer Entropy, Effective Transfer Entropy, and Normalized Transfer Entropy matrices for the main block, together with the stocks belonging to each country in crisis, for each country separately, are calculated using the same techniques described in the last section. Coordinates are associated to each stock, with the reassignments slightly changing the positions of the original stocks. Figure 21 shows the stocks belonging to each country in crisis (black dots) and the stocks belonging to the other countries (white dots) in such a way that their distances represent the approximate smallest value of the Normalized Transfer Entropy between nodes i and j (we choose, as before, the smallest distance between D_{ij} and D_{ji}). The nodes corresponding to the lagged stocks are not represented in the graph, and the connections between stocks are also not shown.

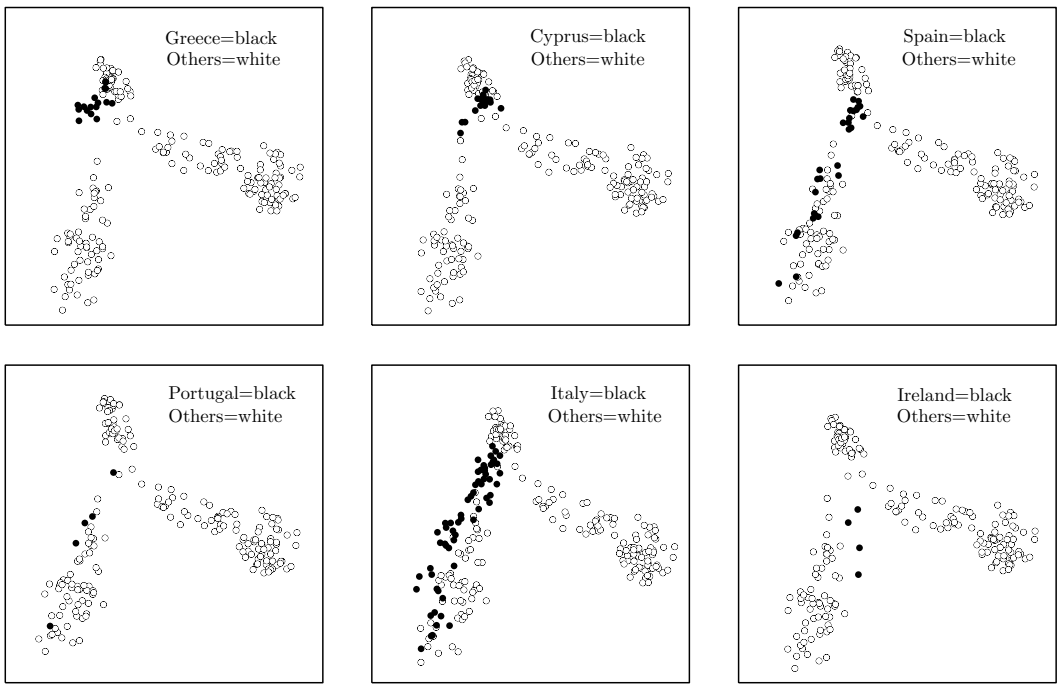

Fig. 21. Two dimensional representations of the stocks as nodes in coordinates that simulate the distances

between them obtained from the ETE, without the lagged nodes. Black dots correspond to stocks belonging to countries in crisis and white dots represent the other stocks. The order of countries is Greece, Cyprus, Spain, Portugal, Italy, and Ireland.

Stocks from Greece and Cyprus occupy positions close to the stocks of Australasia, probably a consequence of the time zones in which those two markets operate. The stocks from Spain and Portugal, Italy and Ireland are scattered along the main European cluster.

Figure 22 shows false color maps of the ETEs from lagged stocks belonging to the countries in crisis to the other stocks of major financial companies. Looking at the ETEs from Greek companies, one can see a medium transfer of entropy from those stocks mainly to stocks of European companies. The ETEs from Cypriot companies are not particularly strong, except for the stocks of some banks, which transfer entropy mainly to stocks from Europe and, particularly, to stocks from Greece. Stocks from Spain also influence mainly stocks of European financial companies. Some Portuguese stocks have large ETEs to European companies and, mainly, to some stocks from Spain. Stocks from Italy have some strong influence on stocks from other European financial companies, and stocks from Ireland have some mild influences on European stocks.

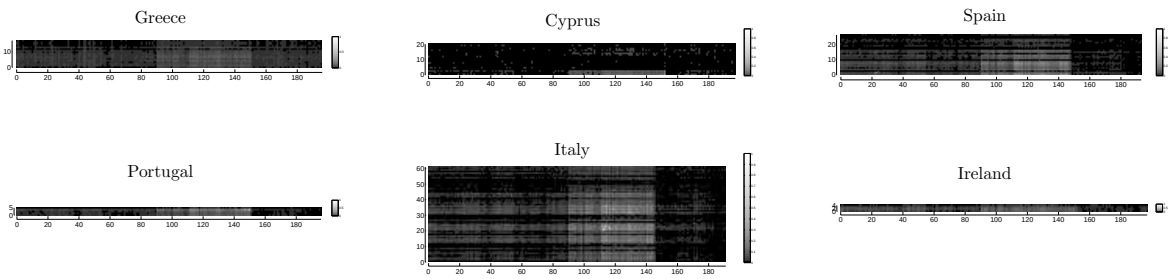

Fig. 22. False color pictures of the ETEs from the lagged selected stocks from countries in crisis to the original stocks of major financial companies. The order of countries is Greece, Cyprus, Spain, Portugal, Italy, and Ireland.

Table 5 shows the first five stocks that receive the most ETE from the stocks of each country in crisis. Almost all stocks that receive the most ETE are banks, with the exception of the ING Groep, which is a Dutch corporation that specializes in general banking services and in insurance, and so is not just an insurance company, but also a bank. The stocks that are most affected by Greek stocks are well spread among European banks, with the most affected one being the ING Grope from the Netherlands. The stock most affected by Cypriot stocks is the one of the National Bank of Greece, what is expected due to the economic and financial relations between Cyprus and Greece. The remaining influence is evenly divided by some other European stocks. The ETE transmitted from Spain to the five most influenced stocks is larger than the ETE transmitted by Greece and Cyprus, and the influence is evenly divided among the European stocks. Portuguese stocks transmit more entropy to two of the largest Spanish banks, and also to some other European stocks. The influence of Italian stocks is much larger than the influence of other stocks belonging to the group of countries in crisis, and it spreads rather evenly among some European stocks. The influence from Irish stocks is low, and evenly distributed among European stocks, including two from the UK.

One must keep in mind that what we are measuring is the sum of ETEs to a particular company, and so the number of companies that send the ETEs is important, but since the number of relevant financial companies a country has is an important factor of its influence, we here consider the sum of ETEs as a determinant of the influence of one country on another.

It is interesting to see that there are some stocks that are consistently more influenced by the stocks of countries in crisis. The Deutsche Bank appears in five lists, and the ING Groep and the KBC Groep appear in four lists. Most of the stocks listed are also some of the more central ones according to different centrality criteria.

Stock	ETE	Country	Industry	Sub-industry
Greece				
ING Groep	1.04	Netherlands	Insurance	Life/Health Insurance
KBC Groep	1.04	Belgium	Banks	Commercial Banks
Deutsche Bank	0.98	Germany	Banks	Diversified Banking Institution
Société Générale	0.98	France	Banks	Diversified Banking Institution
Crédit Agricole	0.94	France	Banks	Diversified Banking Institution
Cyprus				
National Bank of Greece	0.68	Greece	Banks	Commercial Banks
KBC Groep NV	0.34	Belgium	Banks	Commercial Banks
Deutsche Bank AG	0.33	Germany	Banks	Diversified Banking Institution
ING Groep NV	0.30	Netherlands	Insurance	Life/Health Insurance
DANSKE DC	0.28	Denmark	Banks	Commercial Banks
Spain				
Deutsche Bank	2.34	Germany	Banks	Diversified Banking Institution
BNP Paribas	2.33	France	Banks	Diversified Banking Institution
AXA	2.31	France	Insurance	Multi-line Insurance
ING Groep	2.21	Netherlands	Insurance	Life/Health Insurance
KBC Groep	2.17	Belgium	Banks	Commercial Bank
Portugal				
Banco Santander	0.91	Spain	Banks	Commercial Bank
Banco Bilbao Vizcaya Argentaria	0.72	Spain	Banks	Commercial Bank
BNP Paribas	0.62	France	Banks	Diversified Banking Institution
Deutsche Bank	0.60	Germany	Banks	Diversified Banking Institution
AXA	0.60	France	Insurance	Multi-line Insurance
Italy				
AXA	6.37	France	Insurance	Multi-line Insurance
Deutsche Bank AG	6.29	Germany	Banks	Diversified Banking Institution
BNP Paribas	6.18	France	Banks	Diversified Banking Institution
Banco Bilbao Vizcaya Argentaria	5.90	Spain	Banks	Commercial Bank
Societe Generale	5.84	France	Banks	Diversified Banking Institution
Ireland				
ING Groep NV	0.39	Netherlands	Insurance	Life/Health Insurance
Barclays	0.37	UK	Banks	Diversified Banking Institution
Lloyds Banking Group	0.37	UK	Banks	Diversified Banking Institution
Aegon NV	0.36	Netherlands	Insurance	Multi-line Insurance
KBC Groep NV	0.36	Belgium	Banks	Commercial Bank

Table 5. Five stocks that receive more ETE from the stocks of each country in crisis. In the table, are shown the name of the company, the total ETE received from the stocks of countries in crisis, the country the stock belongs to, the industry and sub-industry.

Table 6 shows the first five stocks that send the most ETE from the stocks of each country in crisis (four, in the case of Ireland). The most influential stocks are mainly those of banks, but we also have highly influent stocks belonging to insurance companies and to investment companies. The influence of Greece is distributed among some banks, and the influence of Cyprus is also mainly distributed among banks. The Spanish influence also comes from commercial banks, and is concentrated on the top three ones. The same applies to Portugal, with the main ETE being transmitted from a stock that belongs to a Spanish bank but that is also negotiated in Portugal. The most influential stocks from Italy are those of companies that are originally from other European countries, but whose stocks are also negotiated in Italy. The influence of Ireland is mainly distributed among two banks and one insurance company.

So we may conclude that the most influenced stocks by stocks of the countries in crisis according to ETE are those of European companies, and mainly some stocks belonging to some particular banks. The stocks that influence the most, also according to the ETE criterium, are those of banks belonging to the countries in crisis, in particular if the banks are native to other countries, but their stocks are negotiated in the country in crisis.

Stock	ETE	Industry	Sub-industry
Greece			
National Bank of Greece	5.95	Banks	Commercial Bank
Piraeus Bank	4.68	Banks	Commercial Bank
Cyprus Popular Bank	4.48	Banks	Commercial Bank
Eurobank Ergasias	4.38	Banks	Commercial Bank
Bank of Cyprus	4.28	Banks	Commercial Bank
Cyprus			
Cyprus Popular Bank	5.18	Banks	Commercial Banks
Bank of Cyprus	4.01	Banks	Commercial Banks
Hellenic Bank	3.02	Banks	Commercial Banks
Interfund Investments	2.12	Investment Companies	Investment Companies
Demetra Investments	1.88	Investment Companies	Investment Companies
Spain			
Banco Santander	15.90	Banks	Commercial Bank
Banco Bilbao Vizcaya Argentaria	14.74	Bank	Commercial Bank
Banco Popular Espanol	11.35	Banks	Commercial Bank
Banco de Sabadell	10.47	Bank	Commercial Bank
Banco Bradesco	9.99	Banks	Commercial Bank
Portugal			
Banco Santander	12.67	Banks	Commercial Banks
Banco Espírito Santo	8.60	Banks	Commercial Banks
Banco BPI	8.32	Banks	Commercial Banks
Banco Comercial Portugues	3.08	Banks	Commercial Banks
Espírito Santo Financial Group	4.08	Banks	Commercial Banks
Italy			
ING Groep NV	15.91	Insurance	Life - Health Insurance
Deutsche Bank AG	15.43	Banks	Diversified Banking Institution
AXA	15.23	Insurance	Multi-line Insurance
BNP Paribas	14.51	Banks	Diversified Banking Institution
UniCredit SpA	14.09	Banks	Diversified Banking Institution
Ireland			
Bank of Ireland	12.67	Banks	Commercial Bank
Permanent TSB Group Holdings	8.60	Insurance	Property - Casualty Insurance
Allied Irish Banks	8.32	Banks	Commercial Bank
FBD Holdings	3.08	Insurance	Property - Casualty Insurance

Table 6. Five stocks that send more ETE from each country in crisis. In the table, are shown the name of the company, the total ETE sent to the stocks of main financial companies, the industry and sub-industry.

6 Conclusions

We have seen in this work how the stocks of the top 197 financial companies, in market volume, relate to one another, using both their correlations and the Transfer Entropy between them. We saw that they are related first by country where the stocks are negotiated, and then by industry and sub-industry. The network structures for correlation and for Transfer Entropy are very different from one another, being the network obtained using Transfer Entropy a directed one, with causal influences between the stocks. The use of original and lagged log-returns also revealed some relationships between stocks, with the stocks of a previous day influencing the stocks of the following day. A study of the centralities of the stocks revealed that the most central ones are those of insurance companies of Europe and of the USA, or of banks of the USA and Europe. Since insurance and reinsurance companies are major CDS (Credit Default Securities) sellers, and banks are both major CDS buyers and sellers, some of this centrality of insurance companies, followed by banks, might be explained by the selling and buying of CDS.

A further study of the causality relations between stocks of companies belonging to countries in crisis, namely Greece, Cyprus, Spain, Portugal, Italy, and Ireland, reveal which are the most affected financial companies belonging to the group of largest financial stocks. This calls attention to liabilities of those companies to possible defaults or fall of stocks prices of companies belonging to those countries in crisis.

This work plants the seeds for the study of contagion among financial institutions, but now based on a real network, showing which companies are most central for the propagation of crises and which ones are more dependent on failing economies. This may be used to develop policies for avoiding the spread of financial crises.

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A List of stocks used

Here are displayed, in order of country and of industry and sub-industry, the stocks that are used in the present work, not considering stocks from particular countries in crisis.

Country	Company	Industry	Sector
USA 1	Bank of America Corp	Banks	Diversified Banking Inst
USA 2	Citigroup Inc	Banks	Diversified Banking Inst
USA 3	Goldman Sachs Group Inc/The	Banks	Diversified Banking Inst
USA 4	JPMorgan Chase & Co	Banks	Diversified Banking Inst
USA 5	Morgan Stanley	Banks	Diversified Banking Inst
USA 6	Comerica Inc	Banks	Super-Regional Banks-US
USA 7	Capital One Financial Corp	Banks	Super-Regional Banks-US
USA 8	KeyCorp	Banks	Super-Regional Banks-US
USA 9	PNC Financial Services Group Inc/The	Banks	Super-Regional Banks-US
USA 10	SunTrust Banks Inc	Banks	Super-Regional Banks-US
USA 11	US Bancorp	Banks	Super-Regional Banks-US
USA 12	Wells Fargo & Co	Banks	Super-Regional Banks-US
USA 13	Fifth Third Bancorp	Banks	Super-Regional Banks-US
USA 14	Huntington Bancshares Inc/OH	Banks	Super-Regional Banks-US
USA 15	BB&T Corp	Banks	Commer Banks-Southern US
USA 16	First Horizon National Corp	Banks	Commer Banks-Southern US
USA 17	Regions Financial Corp	Banks	Commer Banks-Southern US
USA 18	M&T Bank Corp	Banks	Commer Banks-Eastern US
USA 19	Zions Bancorporation	Banks	Commer Banks-Western US
USA 20	Bank of New York Mellon Corp/The	Banks	Fiduciary Banks
USA 21	State Street Corp	Banks	Fiduciary Banks
USA 22	Northern Trust Corp	Banks	Fiduciary Banks
USA 23	Banco Bradesco SA	Banks	Commer Banks Non-US
USA 24	Itau Unibanco Holding SA	Banks	Commer Banks Non-US
USA 25	Banco Santander Chile	Banks	Commer Banks Non-US
USA 26	Credicorp Ltd	Banks	Commer Banks Non-US
USA 27	American Express Co	Diversified Finan Serv	Finance-Credit Card
USA 28	Ameriprise Financial Inc	Diversified Finan Serv	Invest Mgmnt/Advis Serv
USA 29	Franklin Resources Inc	Diversified Finan Serv	Invest Mgmnt/Advis Serv
USA 30	BlackRock Inc	Diversified Finan Serv	Invest Mgmnt/Advis Serv
USA 31	Invesco Ltd	Diversified Finan Serv	Invest Mgmnt/Advis Serv
USA 32	Legg Mason Inc	Diversified Finan Serv	Invest Mgmnt/Advis Serv
USA 33	T Rowe Price Group Inc	Diversified Finan Serv	Invest Mgmnt/Advis Serv
USA 34	E*TRADE Financial Corp	Diversified Finan Serv	Finance-Invest Bnkr/Brkr
USA 35	IntercontinentalExchange Inc	Diversified Finan Serv	Finance-Other Services
USA 36	NYSE Euronext	Diversified Finan Serv	Finance-Other Services
USA 37	NASDAQ OMX Group Inc/The	Diversified Finan Serv	Finance-Other Services
USA 38	Hudson City Bancorp Inc	Savings & Loans	S& L/Thrifts-Eastern US
USA 39	People's United Financial Inc	Savings & Loans	S& L/Thrifts-Eastern US
USA 40	ACE Ltd	Insurance	Multi-line Insurance
USA 41	American International Group Inc	Insurance	Multi-line Insurance
USA 42	Assurant Inc	Insurance	Multi-line Insurance
USA 43	Allstate Corp/The	Insurance	Multi-line Insurance
USA 44	Genworth Financial Inc	Insurance	Multi-line Insurance
USA 45	Hartford Financial Services Group Inc	Insurance	Multi-line Insurance
USA 46	Loews Corp	Insurance	Multi-line Insurance
USA 47	MetLife Inc	Insurance	Multi-line Insurance
USA 48	XL Group PLC	Insurance	Multi-line Insurance
USA 49	Cincinnati Financial Corp	Insurance	Multi-line Insurance
USA 50	Principal Financial Group Inc	Insurance	Life/Health Insurance
USA 51	Lincoln National Corp	Insurance	Life/Health Insurance
USA 52	Aflac Inc	Insurance	Life/Health Insurance
USA 53	Torchmark Corp	Insurance	Life/Health Insurance
USA 54	Unum Group	Insurance	Life/Health Insurance
USA 55	Prudential Financial Inc	Insurance	Life/Health Insurance
USA 56	Travelers Cos Inc/The	Insurance	Property/Casualty Ins

Country	Company	Industry	Sector
USA 57	Chubb Corp/The	Insurance	Property/Casualty Ins
USA 58	Progressive Corp/The	Insurance	Property/Casualty Ins
USA 59	Aon PLC	Insurance	Insurance Brokers
USA 60	Marsh & McLennan Cos Inc	Insurance	Insurance Brokers
USA 61	Berkshire Hathaway Inc	Insurance	Reinsurance
USA 62	CBRE Group Inc	Real Estate	Real Estate Mgmnt/Servic
USA 63	Apartment Investment & Management Co	REITS	REITS-Apartments
USA 64	AvalonBay Communities Inc	REITS	REITS-Apartments
USA 65	Equity Residential	REITS	REITS-Apartments
USA 66	Boston Properties Inc	REITS	REITS-Office Property
USA 67	Host Hotels & Resorts Inc	REITS	REITS-Hotels
USA 68	Prologis Inc	REITS	REITS-Warehouse/Industr
USA 69	Public Storage	REITS	REITS-Storage
USA 70	Simon Property Group Inc	REITS	REITS-Regional Malls
USA 71	Macerich Co/The	REITS	REITS-Regional Malls
USA 72	Kimco Realty Corp	REITS	REITS-Shopping Centers
USA 73	Ventas Inc	REITS	REITS-Health Care
USA 74	HCP Inc	REITS	REITS-Health Care
USA 75	Health Care REIT Inc	REITS	REITS-Health Care
USA 76	American Tower Corp	REITS	REITS-Diversified
USA 77	Weyerhaeuser Co	REITS	REITS-Diversified
USA 78	Vornado Realty Trust	REITS	REITS-Diversified
USA 79	Plum Creek Timber Co Inc	REITS	REITS-Diversified
Canada 1	Bank of Montreal	Banks	Commer Banks Non-US
Canada 2	Bank of Nova Scotia	Banks	Commer Banks Non-US
Canada 3	Canadian Imperial Bank of Commerce/Canada	Banks	Commer Banks Non-US
Canada 4	National Bank of Canada	Banks	Commer Banks Non-US
Canada 5	Royal Bank of Canada	Banks	Commer Banks Non-US
Canada 6	Toronto-Dominion Bank/The	Banks	Commer Banks Non-US
Canada 7	Manulife Financial Corp	Insurance	Life/Health Insurance
Canada 8	Power Corp of Canada	Insurance	Life/Health Insurance
Canada 9	Sun Life Financial Inc	Insurance	Life/Health Insurance
Canada 10	Brookfield Asset Management Inc	Real Estate	Real Estate Oper/Develop
Chile	Banco de Chil	Banks	Commer Banks Non-US
UK 1	Barclays PLC	Banks	Diversified Banking Inst
UK 2	HSBC Holdings PLC	Banks	Diversified Banking Inst
UK 3	Lloyds Banking Group PLC	Banks	Diversified Banking Inst
UK 4	Royal Bank of Scotland Group PLC	Banks	Diversified Banking Inst
UK 5	Standard Chartered PLC	Banks	Commer Banks Non-US
UK 6	Aberdeen Asset Management PLC	Diversified Finan Serv	Invest Mgmnt/Advis Serv
UK 7	Man Group PLC	Diversified Finan Serv	Invest Mgmnt/Advis Serv
UK 8	Schroders PLC	Diversified Finan Serv	Invest Mgmnt/Advis Serv
UK 9	Old Mutual PLC	Diversified Finan Serv	Invest Mgmnt/Advis Serv
UK 10	Provident Financial PLC	Diversified Finan Serv	Finance-Consumer Loans
UK 11	London Stock Exchange Group PLC	Diversified Finan Serv	Finance-Other Services
UK 12	Aviva PLC	Insurance	Life/Health Insurance
UK 13	Legal & General Group PLC	Insurance	Life/Health Insurance
UK 14	Prudential PLC	Insurance	Life/Health Insurance
UK 15	Standard Life PLC	Insurance	Life/Health Insurance
UK 16	RSA Insurance Group PLC	Insurance	Property/Casualty Ins
UK 17	3i Group PLC	Private	Private
UK 18	Hammerson PLC	REITS	REITS-Shopping Centers
UK 19	British Land Co PLC	REITS	REITS-Diversified
UK 20	Land Securities Group PLC	REITS	REITS-Diversified
UK 21	Segro PLC	REITS	REITS-Diversified
France 1	Credit Agricole SA	Banks	Diversified Banking Inst
France 2	BNP Paribas SA	Banks	Diversified Banking Inst
France 3	Societe Generale SA	Banks	Diversified Banking Inst
France 4	AXA SA	Insurance	Multi-line Insurance
Germany 1	Commerzbank AG	Banks	Commer Banks Non-US
Germany 2	Deutsche Bank AG	Banks	Diversified Banking Inst
Germany 3	Deutsche Boerse AG	Diversified Finan Serv	Finance-Other Services
Germany 4	Allianz SE	Insurance	Multi-line Insurance
Germany 5	Muenchener Rueckversicherungs AG	Insurance	Reinsurance
Switzerland 1	Credit Suisse Group AG	Banks	Diversified Banking Inst
Switzerland 2	UBS AG	Banks	Diversified Banking Inst
Switzerland 3	GAM Holding AG	Diversified Finan Serv	Invest Mgmnt/Advis Serv
Switzerland 4	Baloise Holding AG	Insurance	Multi-line Insurance
Switzerland 5	Zurich Insurance Group AG	Insurance	Multi-line Insurance
Switzerland 6	Swiss Life Holding AG	Insurance	Life/Health Insurance
Switzerland 7	Swiss Re AG	Insurance	Reinsurance
Austria	Erste Group Bank AG	Banks	Commer Banks Non-US

Country	Company	Industry	Sector
Netherlands 1	Aegon NV	Insurance	Multi-line Insurance
Netherlands 2	ING Groep NV	Insurance	Life/Health Insurance
Belgium 1	KBC Groep NV	Banks	Commer Banks Non-US
Belgium 2	Ageas	Insurance	Life/Health Insurance
Sweden 1	Nordea Bank AB	Banks	Commer Banks Non-US
Sweden 2	Skandinaviska Enskilda Banken AB	Banks	Commer Banks Non-US
Sweden 3	Svenska Handelsbanken AB	Banks	Commer Banks Non-US
Sweden 4	Swedbank AB	Banks	Commer Banks Non-US
Sweden 5	Investor AB	Investment Companies	Investment Companies
Denmark	Danske Bank A/S	Banks	Commer Banks Non-US
Finland	Sampo	Insurance	Multi-line Insurance
Norway	DNB ASA	Banks	Commer Banks Non-US
Italy 1	Banca Monte dei Paschi di Siena SpA	Banks	Commer Banks Non-US
Italy 2	Intesa Sanpaolo SpA	Banks	Commer Banks Non-US
Italy 3	Mediobanca SpA	Banks	Commer Banks Non-US
Italy 4	Unione di Banche Italiane SCPA	Banks	Commer Banks Non-US
Italy 5	UniCredit SpA	Banks	Diversified Banking Inst
Italy 6	Assicurazioni Generali SpA	Insurance	Multi-line Insurance
Spain 1	Banco Bilbao Vizcaya Argentaria SA	Banks	Commer Banks Non-US
Spain 2	Banco Popular Espanol SA	Banks	Commer Banks Non-US
Spain 3	Banco de Sabadell SA	Banks	Commer Banks Non-US
Spain 4	Banco Santander SA	Banks	Commer Banks Non-US
Portugal	Banco Espírito Santo SA	Banks	Commer Banks Non-US
Greece	National Bank of Greece SA	Banks	Commer Banks Non-US
Japan 1	Shinsei Bank Ltd	Banks	Commer Banks Non-US
Japan 2	Mitsubishi UFJ Financial Group Inc	Banks	Diversified Banking Inst
Japan 3	Sumitomo Mitsui Trust Holdings Inc	Banks	Commer Banks Non-US
Japan 4	Sumitomo Mitsui Financial Group Inc	Banks	Commer Banks Non-US
Japan 5	Mizuho Financial Group Inc	Banks	Commer Banks Non-US
Japan 6	Credit Saison Co Ltd	Diversified Finan Serv	Finance-Credit Card
Japan 7	Daiwa Securities Group Inc	Diversified Finan Serv	Finance-Invest Bnkr/Brkr
Japan 8	Nomura Holdings Inc	Diversified Finan Serv	Finance-Invest Bnkr/Brkr
Japan 9	ORIX Corp	Diversified Finan Serv	Finance-Leasing Compan
Japan 10	Tokio Marine Holdings In	Insurance	Property/Casualty Ins
Japan 11	Mitsui Fudosan Co Ltd	Real Estate	Real Estate Oper/Develop
Japan 12	Mitsubishi Estate Co Ltd	Real Estate	Real Estate Mgmnt/Servic
Japan 13	Sumitomo Realty & Development Co Ltd	Real Estate	Real Estate Oper/Develop
Hong Kong 1	Hang Seng Bank Ltd	Banks	Commer Banks Non-US
Hong Kong 2	Industrial & Commercial Bank of China Ltd	Banks	Commer Banks Non-US
Hong Kong 3	BOC Hong Kong Holdings Ltd	Banks	Commer Banks Non-US
Hong Kong 4	China Construction Bank Corp	Banks	Commer Banks Non-US
Hong Kong 5	Hong Kong Exchanges and Clearing Ltd	Diversified Finan Serv	Finance-Other Services
Hong Kong 6	Ping An Insurance Group Co of China Ltd	Insurance	Multi-line Insurance
Hong Kong 7	China Life Insurance Co Ltd	Insurance	Life/Health Insurance
Hong Kong 8	Cheung Kong Holdings Ltd	Real Estate	Real Estate Oper/Develop
Hong Kong 9	Sun Hung Kai Properties Ltd	Real Estate	Real Estate Oper/Develop
South Korea	Shinhan Financial Group Co Ltd	Diversified Finan Serv	Diversified Finan Serv
Taiwan	Cathay Financial Holding Co Ltd	Insurance	Life/Health Insurance
Singapore 1	DBS Group Holdings Ltd	Banks	Commer Banks Non-US
Singapore 2	Oversea-Chinese Banking Corp Ltd	Banks	Commer Banks Non-US
Singapore 3	United Overseas Bank Ltd	Banks	Commer Banks Non-US
Australia 1	Australia & New Zealand Banking Group Ltd	Banks	Commer Banks Non-US
Australia 2	Commonwealth Bank of Australia	Banks	Commer Banks Non-US
Australia 3	National Australia Bank Ltd	Banks	Commer Banks Non-US
Australia 4	Westpac Banking Corp	Banks	Commer Banks Non-US
Australia 5	Macquarie Group Ltd	Diversified Finan Serv	Finance-Invest Bnkr/Brkr
Australia 6	ASX Ltd	Diversified Finan Serv	Finance-Other Services
Australia 7	AMP Ltd	Insurance	Life/Health Insurance
Australia 8	Suncorp Group Ltd	Insurance	Life/Health Insurance
Australia 9	Insurance Australia Group Ltd	Insurance	Property/Casualty Ins
Australia 10	QBE Insurance Group Ltd	Insurance	Property/Casualty Ins
Australia 11	Lend Lease Group	Real Estate	Real Estate Mgmnt/Servic
Australia 12	CFS Retail Property Trust Group	REITS	REITS-Shopping Centers
Australia 13	Westfield Group	REITS	REITS-Shopping Centers
Australia 14	Dexus Property Group	REITS	REITS-Diversified
Australia 15	Goodman Group	REITS	REITS-Diversified
Australia 16	GPT Group	REITS	REITS-Diversified
Australia 17	Mirvac Group	REITS	REITS-Diversified
Australia 18	Stockland	REITS	REITS-Diversified

References

Allen, F. and Gale, D. (2000). Financial contagion. Journal of Political Economy 108, 1-33.

Allen, F. and Babus, A. (2009). Networks in finance. In Kleindorfer, P., Wing, Y.,and Gunther, R. (eds.), The network challenge: strategy, profit, and risk in an interlinked world. Wharton School Publishing.

Ausloos, M. and Lambiotte, R. (2007). Clusters or networks of economies? A macroeconomy study through gross domestic product. Physica A 382, 16-21.

Borg, I. and Groenen, P. (2005). Modern Multidimensional Scaling: theory and applications. 2nd edition, Springer-Verlag.

Dimp, T. Huergo, L. and Peter, F.J. (2012). Using transfer entropy to measure information flows from and to the CDS market. Midwest Finance Association 2012 Annual Meetings Paper.

Haldane, A. (2009). Rethinking the financial network. Speech delivered at the Financial Student Association, Amsterdam, Aprol.

Jizba, P., Kleinert, H. and Shefaat, M. (2012). Renyi's information transfer between financial time series. Physica A 391, 2971-2989.

Kwon, O. and Yang, J-S (2008a). Information flow between composite stock index and individual stocks. Physica A 387, 2851-2856.

Kwon, O. and Yang, J-S (2008b). Information flow between stock indices. European Physics Letters 82, 68003.

Mantegna, R.N. (1999). Hierarchical structure in financial markets. The European Physics Journal B 11, 193.

Newman, M.E.J. (2010). Networks, and introduction. Oxford University Press.

Onnela, J.-P., Chakraborti, A., Kaski, K., and Kertész, J. (2002). Dynamic asset trees and portfolio analysis. The European Physics Journal B 30, 285-288.

Onnela, J.-P., Chakraborti, A., and Kaski, K. (2003). Dynamics of market correlations: taxonomy and portfolio analysis. Physical Review E 68, 1-12.

Onnela, J.-P., Chakraborti, A., Kaski, K., and Kertész, J. (2003). Dynamic asset trees and Black Monday. Physica A 324, 247-252.

Onnela, J.-P., Chakraborti, A., Kaski, K., Kertész, J., and Kanto, A. (2003). Asset trees and asset graphs in financial markets. Physica Scripta T 106, 48-54.

Onnela, J.-P., Chakraborti, A., Kaski, K., and Kertész, J. (2004). Clustering and information in correlation based financial networks. The European Physics Journal B 38, 353-362.

Sandoval Jr., L. (2012a). To lag or not to lag? How to compare indices of stock markets that operate at different times. arXiv:1201.4586.

Sandoval Jr., L. (2012b). A Map of the Brazilian Stock Market. Advances in Complex Systems 15, 1250042-1250082.

Sandoval Jr., L. (2013). Cluster formation and evolution in networks of financial market indices. Algorithmic Finance 2, 3-43.

Schreiber, T. (2000). Measuring information transfer. Physical Review Letters 85, 461-464.

Shannon, C.E. (1948). A Mathematical Theory of Communication. Bell System Technical Journal 27, 379–423, 623-656.

Sinha, S. and Pan R.K. (2007). Uncovering the internal structure of the Indian financial market: cross-correlation behavior in the NSE. In "Econophysics of markets and business networks", Springer, 215-226.

Upper, C. (2011). Simulation methods to assess the danger of contagion in interbank markets. Journal of Financial Stability 7 (3), 111–125.