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ABSTRACT

This paper studies the usefulness of factors embedded on the common movements of exchange rates in forecasting the exchange rate Real/Dollar. The results show that considering the entire period of the sample from January 1999 to August 2011, no one model containing the factors is able to beat a random walk model. However, when the period directly following the adoption of the floating exchange rate regime is discarded, there is evidence that several models containing these factors beat the random walk. Lastly, the paper shows that the addition of factors improves the predictive power of the models comprising only macroeconomic variables commonly used in the literature to forecast the exchange rate.

JEL Classification

F31; F47

Keywords

Exchange rates; Factor models; Out-of-sample forecasting

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1. Introduction

Following the collapse of Bretton Woods at the beginning of the 1970s and the adoption of floating exchange rate regimes by most developed countries, the behavior of exchange rates has been a frequent topic of analysis as this variable exerts a significant impact on countries' trade balance, price levels and output. Considering the role of the exchange rate on the economy, its forecasting is of great relevance. Unfortunately, the body of research dedicated to analyzing the predictive power of models in exchange rate determination has been unable to forecast the exchange rate, especially for short term predictions. This difficulty is considered one of the major weaknesses of international macroeconomics (Bacchetta and Wincoop, 2006).

Meese and Rogoff (1983), in their very influential work, verified the lack of predictive power of theoretical exchange rate models. They argued that little or no information about the future movement of exchange rates over short horizons can be extracted from macroeconomic variables such as monetary aggregates, price levels, output gap or interest rates. They found that no model was able to forecast significantly better than a simple random walk model. After more than 20 years, Cheung, Chinn and Pascual (2005) performed a similar exercise incorporating models developed during the 1990s and applying new econometric techniques. Cheung, Chinn and Pascual concluded that some models perform well for certain projections or specific exchange rates; however, their results do not identify a model that is broadly consistent.[2]

Mark (1995) and Chinn and Meese (1995) are among the authors that have achieved relative success in forecasting the exchange rate. These authors showed that the difference between the exchange rate and the value predicted by a monetary model can be used to predict exchange rate movements over a four-year horizon, outperforming the random walk in an out-of-sample analysis using data from 1973 to 1991. But, as noted by Faust *et.al.* (2003), these results are sensitive to the chosen horizons and sample periods.[3]

One possible explanation to this fragility in forecasting the exchange rate would be how the exchange rate is determined. If the exchange rate is determined by the expected present discounted

[2] The Brazilian experience mirrors the international experience. Muinhos, Alves and Riella (2003) found that models incorporating uncovered interest rate parity outperformed the random walk model when describing the behavior of the Brazilian exchange rate from May 1999 to December 2001. Yet, Moura, Lima and Mendonça (2008), from January 1999 to December 2007, models that incorporate the Taylor rule outperformed the random walk model.

[3] The use of panel techniques to forecast exchange rates has also been reaching relative success. Applying various techniques, Mark and Sul (2001) and Groen (2005) used panels from various countries to determine the co-integration relationship between exchange rates and monetary fundamentals and used this relationship to successfully forecast the exchange rate over long horizons.

value of current and future fundamentals it is possible that the evolution of the exchange rate is affected not only by the dynamics of observables fundamentals like monetary aggregates, price level or output but also by unobservable variables such as risk premium or noise trading. As discussed by Engle, Mark and West (2008), if these unobservable factors have little correlation with the observable, it reduces the predictive power of the models, leading to the weak results found by the literature.[4]

This paper contributes to the literature that attempts to analyze the ability of macroeconomic models to forecast the exchange rate by using a model that, in addition to the traditional macroeconomic variables, contains factors that are extracted from the dynamics of the exchange rate of a set of 19 countries. The paper analyzes whether the use of these factors is valuable in forecasting the exchange rate of the Brazilian Real to the US Dollar after the adoption the floating exchange rate regime in Brazil.

The use of factor models might be helpful in forecasting the exchange rate if (1) the information embedded in the common movements of the exchange rates of various countries is related to the unobservable variables and (2) these variables play a significant role in the dynamics of the exchange rate. The advantage of factor models compared with other models is that they improve the predictive power of the models without losing their practicality; they allow the information set to be condensed in a small number of factors, and these data are easily obtained.

Factor models are widely used to forecast macroeconomic variables (see Forni and Reichlin 1998, Forni et al. 2000, Stock and Watson 2002 and Bai 2004), but they are rarely used in the case of exchange rates. Groen (2006) uses a dynamic factors model to identify the "exchange rate level dictated by fundamentals" and used the difference between this value and the exchange rate to successfully forecast the exchange rate over a two-year horizon. Engel, Mark and West (2008) constructed factors derived from the exchange rates of 17 countries and used these factors to forecast the exchange rate over a two-to-four-year horizon. The authors obtained satisfactory results for the period between 1999 and 2007.

[4]Another explanation supplied by Engel and West (2005) is that whether the exchange rate is determined by the present value deducted by future fundamentals and if at least one of the fundamentals possesses a unit root and the discount factor is near 1, the exchange rate will behave similarly to the random walk. They argue that within this framework, it would be very difficult for macroeconomic models to beat a random walk in forecasting the exchange rate. The authors argue that although unobservable shocks might play a role in the failure of the models, they consider it implausible.

The results indicate that when an out-of-sample forecast exercise is performed using the entire sample period from January 1999 to August 2011, no one model using factors with or without macroeconomic variables is able to beat the random walk model in forecasting the exchange rate. More favorable results are found when we perform the analysis in different periods. When data from the first two years following the adoption of the floating exchange rate regime is removed from the sample there is strong evidence that models that include the factors and macroeconomic variables outperform the random walk model.

Finally, the results in the paper confirm the usefulness of using factor models. The paper shows that the inclusion of the factors systematically increases the predictive power of the models that contain only the macroeconomic variables traditionally used to forecast the exchange rate. This indicates that the factors carry useful information for forecasting the exchange rate and in certain periods are crucial to understanding the dynamics of the exchange rate.

The present study is organized as follows: the following section shows the data and describes the characteristics of a factor model. Section 3 describes the forecasting methodology, section 4 presents the results and section 5 concludes.

2. Data and Methodology

2.1 Data

In this study, monthly data for the floating exchange period in Brazil from January 1999 to August 2011are used to evaluate the models and construct the factors. The exchange rates used are from countries with *de facto* floating exchange rate regimes and independent monetary policies, according to the International Monetary Fund classification. The following countries and economic alliances met the selection criteria: Australia, Brazil, Canada, Chile, South Korea, the Philippines, England, Iceland, Israel, Japan, Mexico, New Zealand, Norway, Poland, South Africa, Sweden, Switzerland, Turkey and the Euro Zone. Exchange rates from the end of the month are used and compared to the U.S. dollar following the convention of local currency quantity per unit of foreign currency.

The Brazilian economic data are obtained from the Brazilian Central Bank (Banco Central do Brasil) and the Brazilian Institute of Geography and Statistics (Instituto Brasileiro de Geografia e Estatística [IBGE]) database. Data for the U.S. are acquired from the Federal Reserve and the Bureau of Labor Statistics indicator databases.

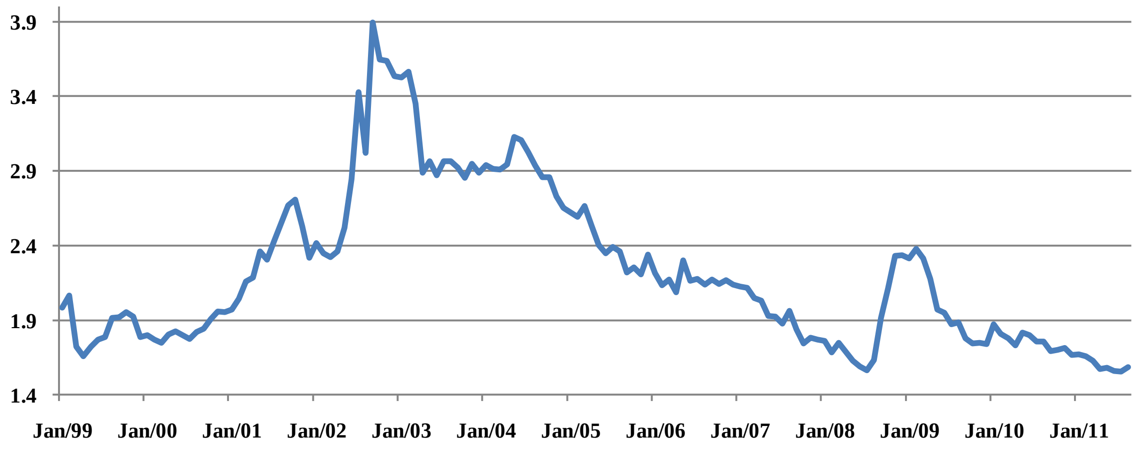
The accumulated inflation over 12 months is used and is based on the Brazilian national consumer price index (Índice Nacional de Preços ao Consumidor [IPCA]) and the Consumer Price Index (CPI) for urban consumers in the U.S. Both indices are seasonally adjusted. Price levels are given by the accumulation of the monthly inflation from the same indices, with a base of 100 in January 1990. Regarding the currency, the monthly M1 is used for both the U.S. and Brazil.

The seasonally adjusted monthly industrial production is used as a proxy for the output, and the output gap is estimated using the Hodrick-Prescott filter. Although the use of industrial production as a proxy for GDP represents only a portion of the total economic activity, this approximation is necessary because the low frequency of the GDP data (quarterly) would significantly reduce the sample size. Given the relatively short history of floating exchange rates in Brazil, the use of monthly data is appropriate.

Figure 1 shows the evolution of the exchange rate of the Brazilian Real to the U.S. Dollar in the period of interest. The trajectory of the exchange rate can be divided into two different periods. From January 1999to 2003, when after a speculative attack, Brazil abandoned a crawling-peg exchange rate regime and adopted a floating exchange rate regime. During this period, the domestic currency depreciated, markedly suffering from several episodes of large devaluation with brief periods of tranquility. Beginning in 2003, there was more stability in the evolution of the exchange rate with a continuous trend of appreciation of the domestic currency only interrupted by the global financial crisis of 2008 that resulted in a rapid and acute devaluation of the Brazilian currency, a movement that was reversed throughout 2009 and 2010.

Figure 1 – Evolution of the Exchange rate Brazilian Real / US$ from 1999 to 2011

Figure 1 shows the trajectory of the exchange rate Real / US$ from January 1999 to August 2011.



2.2 Methodology

An *n*-factor model is used where *n* is given by a cut-off rule. This rule states that 90% of the variance of the series must be cumulatively explained by *n* factors.

For the exchange rate in question, the model is given by the following, as in Engel, Mark and West (2008):

$$s_t = c + \sum_{j=1}^{n} \delta_j f_{jt} + v_t \quad (1)$$

$$s_t = c + F_t + v_t \quad (2)$$

where v_t is a non-correlated shock and c is a constant.

Each factor is considered to be an unobservable variable at one degree of integration. Co-integration between factors is assumed and implies that F_{it}-s_{it} is stationary. The factors, which are not correlated by design, are normalized to yield a mean of 0 and a standard deviation of 1.

The factors in this study are constructed from the exchange rates and are used to capture the dynamics of the exchange rate that are not well explained by observable and measurable economic fundamentals.

In the first stage of the study, a factor analysis is performed on the log-levels of 19 exchange rates for the complete temporal sample. The sample variance explained by each factor is obtained, and the *n* factor cut-off rule is used (i.e., at least 90% of the total variability is explained by the *n* factor).

In the second stage, the *n* factors of the model are estimated using the maximum likelihood method. This process produces temporal series for the estimates of the factors $\hat{f}_{jt}$, j=1,..., n and for the loadings of each factor for the currency in question; that is, $\hat{\delta}_j$, j=1,…, n. Therefore, an “unobservable economic fundamental” is defined as $\hat{F}_t$.

The model is estimated using the follow equation:

$$s_{t+h} - s_t = \alpha + \beta(\hat{F}_t - s_t) + \gamma(z_t - s_t) + u_{t+h} \quad (3)$$

where z_t represents a measure of central tendency obtained through the observable variables. This model considers different economic fundamentals for each specification of z_t.

In the first specification, it is assumed that $\gamma = 0$; that is, the measure of utilized central tendency is given only by the factors obtained through the set of information present in the exchange rates of the 19 countries.

In the second specification, a Taylor rule is used. The z_t in this case is given by:

$$z_t = 1.5(\pi_t - \pi_t^*) + 0.5(\tilde{y}_t - \tilde{y}_t^*) + s_t \quad (4)$$

where π=inflation, $\tilde{y}$=output gap (approximated by the industrial production gap), and variables with * indicate values from the reference country, the United States.

The model that follows the Taylor rule is developed based on the view that interest rates, not monetary aggregates, are the instruments conducting monetary policy. More details on this model can be found in Mark (2008) and Engel and West (2006).

In a third specification,z_t is described according to the following monetary model:

$$z_t = (m_t - m_t^*) - (y_t - y_t^*) \quad (5)$$

where m = log (m1) andy= log (product)approximated by the log of industrial production.

In the final specification, z_t is expressed by the following purchasing power parity model:

$$z_t = (p_t - p_t^*) \quad (6)$$

where p = log (price level).

For the last three models, there is an extensive body of literature related to the prediction of exchange rates using economic fundamentals. For a more comprehensive description of the models above, see Sarno and Taylor (2002).

3. Estimation

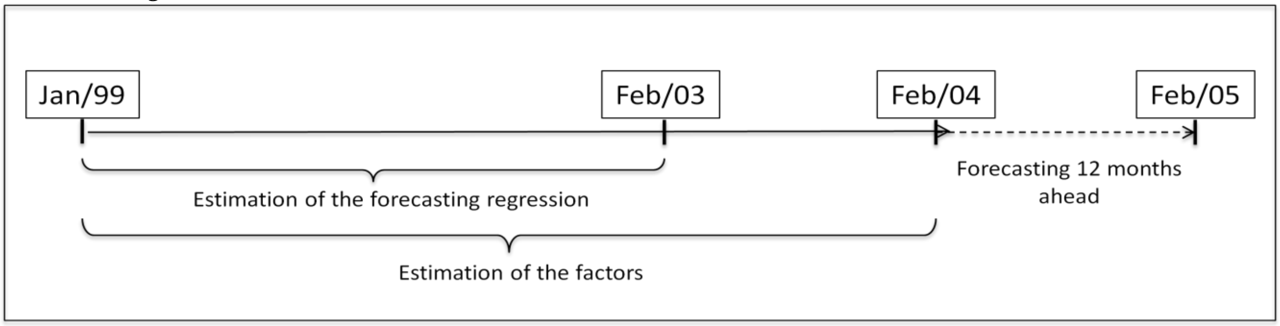
Initially, cointegration tests are performed for all models used throughout the text. The results indicating the existence of cointegration among the variables is showed in the appendix.

3.1 Out-of-sample forecasting

Out-of-sample forecasting is used, following Engel and West (2008) methods, and can be separated into five stages. Figure 2 illustrates the out-of-sample forecasting process predicting the dynamics for the 12-month horizon from February 2004.

Figure 2 – Schematic diagram of the process of out-of-sample forecasting

Figure 2 illustrates the forecasting process. For a specific date (*t*), all of the information available up to that date is used to estimate the factors. The forecasting regression is estimated using the information available until (*t*-*h*), where *h* is the forecasting horizon in months. The forecast regression is used with the information available at *t* to generate a forecast for the exchange rate at *t*+*h*.



The process of out-of-sample forecasting is performed in the following stages. First, estimates of the factors matrix ($\hat{F}$) and its loadings ($\hat{\delta}$) were obtained from the historical series of the exchange rate of the 19 countries, using the maximum likelihood method, as follows:

$$(s_{it} - \mu) = \hat{\delta}\widehat{F_t} + \varepsilon_t \tag{7}$$

After estimating the factors, the exchange rate given by the models is obtained from the following equation (estimated):

$$\bar{S_t} = \hat{\alpha} + X_t\widehat{\Pi} \tag{8}$$

where X_t varies for each specification.

The error-correction equation is then estimated, given by the following:

$$s_{t+h} - s_t = \phi(\overline{S_t} - s_t) + u_t \tag{9}$$

With all of the estimates, the forecast is calculated for the exchange rate, using the following equation:

$$\text{Forecast}(s_{t+h} - s_t) = \hat{\phi}(\overline{S_t} - s_t) \tag{10}$$

Then, another observation is added, and the process is repeated.

In this study, the estimation of the factors covered the period from January 1999 to December 2003, whereas the regression estimation covered the period from January 1999 to December 2002.

One by one, each observation is included in the sample such that the sample available for regression estimation and the factors increases. The maximum likelihood method is used for estimation. This process is repeated for different values of h (different horizons) and different values of X_t (different specifications).

For a specific reference date, the same factors and fundamentals are used; that is, $(\widehat{F}_t - s_t)$ and X_t – remain independent of the forecast horizon h. The regression coefficients vary with h, meaning that the larger the forecast horizon, the smaller the sample size available for regression coefficient estimation.

The forecast statistics for various distinct periods are calculated for the exchange rate Brazilian Real / US$. The sampling period covers January 1999 to August 2011. The five other subsamples excludes the initial period of the floating exchange rate regime in Brazil. The first subsample covers the period from January 2000 to August 2011, the second subsample covers the period from January 2001 to August 2011 and so on. The forecast horizons are always 1, 3, 6 and 12 months into the future.

The out-of-sample forecasting (described above) is then compared with a forecast model that assumes that the exchange rate follows a random walk without a trend; that is:

$$s_{t+h} = s_t \tag{11}$$

The sample root mean square prediction error (RMSPE) is used as the criterion for judging each model. The Theil's U statistic is calculated from the ratio between the RMSPE of each model and the RMSPE of the random walk. U-statistics with a value less than 1 indicate that the model possesses a smaller RMSPE than the random walk model. However, even a value of 1 can be considered evidence against the random walk model. As argued in Clark and West (2006, 2007), if the process generating the exchange rate is, in fact, a random walk, the inclusion of other variables should introduce noise in the forecasting process, leading, on average, to a greater root mean square prediction error than the random walk (and thus producing U-statistics with a value greater than 1).

The hypothesis to be tested is that the models discussed in this study have a smaller RMSPE than the random walk model. This hypothesis can be expressed as follows:

$$\mathrm{H_0}: RMSPE(\mathrm{model_i}) = RMSPE(\text{random walk})$$

$$\mathrm{H_A}: RMSPE(\mathrm{model_i}) < RMSPE(\text{random walk})$$

Where i =1,…,4 and represents each specification of the model.

The following two statistics are presented as the evaluation criteria for forecast quality: Theil's U (TU) and the Clark and West statistic (2006, 2007) (CW). The Clark and West statistic (2006, 2007) is more appropriate for asymptotic tests than the one given by Diebold and Mariano (1995) and West (1996) (DMW) for nested models. As observed by Clark and West (2006, 2007), in nested models, the DMW statistics yields a test statistic with a non-normal distribution, which underestimates the quantity of the null hypothesis rejections.

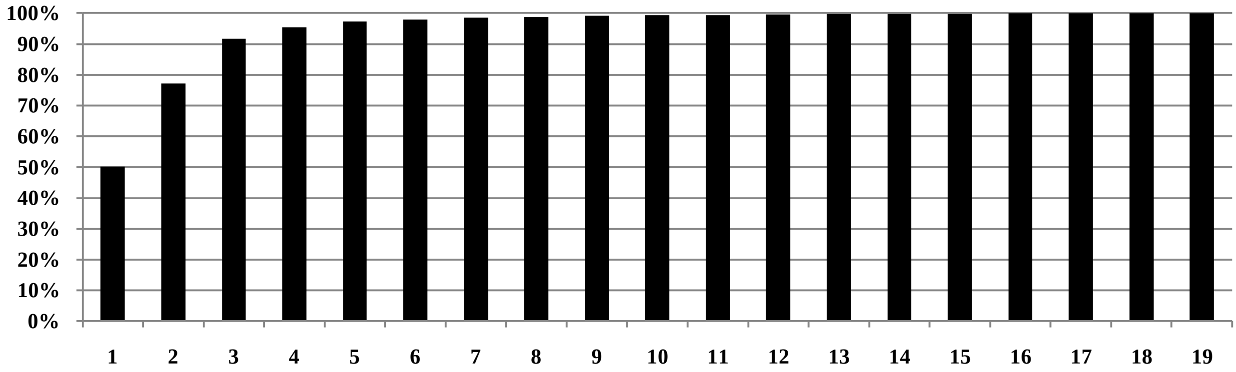
4. Results

4.1 Factor analysis

Figure 3 shows the percentage of the variance of the exchange rate that is cumulative variance explained by the first *n* factors for the complete sample (1999–2011). The figure indicates that three factors jointly explained approximately 92% of the variability in the data.

Figure 3 – Analysis of the factors

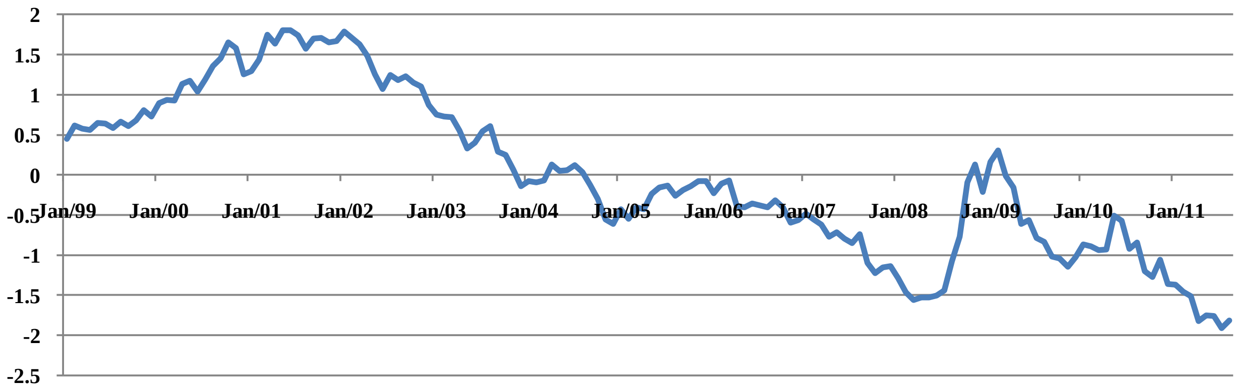
Figure 3 shows cumulative variance explained by the first *n* factors for the complete sample (1999–2011). Factors are estimated by maximum likelihood with n = 19 factors. The complete sample, consisting of 19 exchange rates and 152 months, is used. The loadings are found in the appendix.



One of the main critics of factor models is that much of the time the estimated factors lack an economic interpretation. To perform an analysis, figure 4 plots the time series for the estimated factor $\widehat{f_1}$ throughout the entire sample period.

Figure 4 – Estimate of factor 1

Figure 4 refers to the estimate of factor f1, as in $s_t = \sum_{j=1}^{n} \delta_j f_{jt} + v_t + c$. The factor is standardized such that it has a mean of 0. The estimate follows the standard case for this study using 152 monthly samples and 19 exchange rates, and the factor model is estimated by maximum likelihood.



The main characteristic of factor 1 is that this factor has a significant loading in almost all of the currencies used. It represents a type of weighted mean for all of the exchange rates (with higher weight for developed countries). This mean can be interpreted as reflecting the periods of weakness or strength of the reference currency or a common shock to money demand in these countries. In other words, factor 1 can be viewed as indicating the strength of all of the other currencies against the U.S. dollar.

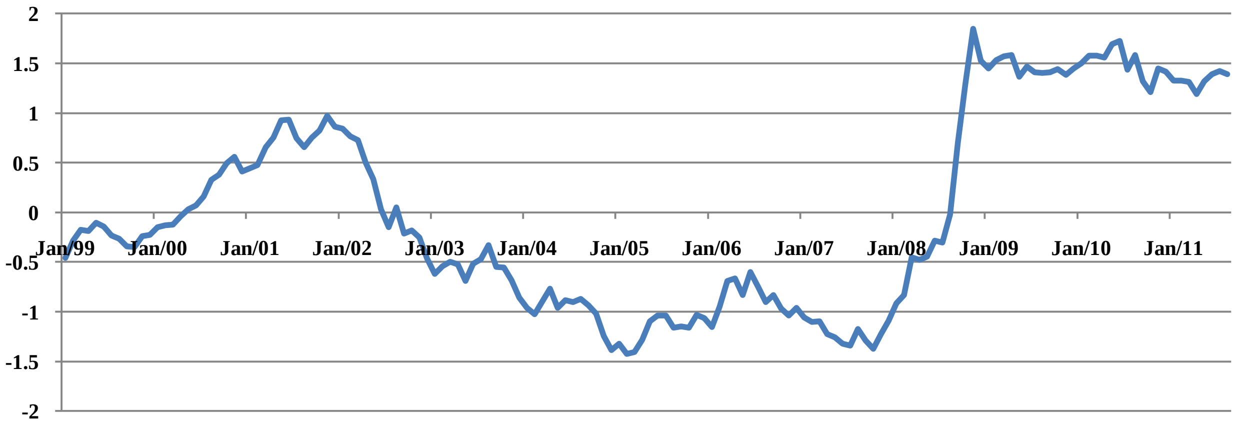
The dynamics of the factor, shown in figure 4, indicate a continuous decrease of the factor between 2001 and 2008 a period when the dollar depreciated; reflecting the flight to quality during the financial crisis there was an increase in the factor in 2008 and 2009 when again there was a reduction in the value of the factor. Importantly, this factor alone explains 50% of the variability of the set of exchange rates.

Although the analysis of the loadings estimates for factor 2 does not give precise information with respect to the interpretation of the factor, the graphical analysis shown in figure 5 that plots the estimate of factor $\widehat{f_2}$ throughout the sample period indicate that this factor may represent investors' risk aversion.

Note that there is a peak in 2001 when events like the September 11thterrorist attack had an impact on investors. This factor has also a break during the second half of 2008, a period that coincides with the onset of the 2008 financial crisis, and after 2008 it continues at close to its maximum value, indicating the continuation of the uncertainties related to the financial turmoil.

Figure 5 – Estimate of factor 2

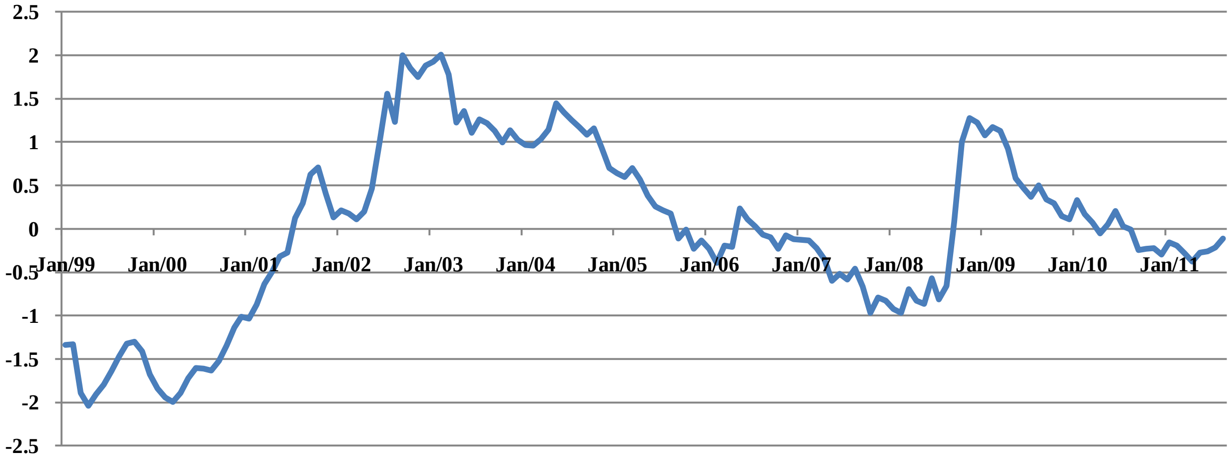
Figure 5 refers to the estimate of factor f2, as in $s_t = \sum_{j=1}^{n} \delta_j f_{jt} + v_t + c$. The factor is standardized such thatit has a mean of 0. The estimate follows the standard case for this work using 152 monthly samples and 19 exchange rates, and the factor model is estimated by maximum likelihood.



The analysis of the loadings of factor 3 and its evolution shown in figure 6, which plots the estimate of factor $\widehat{f}_3$ throughout the entire sample period, indicate that this factor might represent a risk premium. The significance of the loadings, especially regarding the exchange rates of developing countries, suggests that factor 3 reflects a risk premium that is much more prominent in countries in their development phase, such as Brazil or Turkey, than in regions with mature economies, such as Switzerland or the Euro Zone.

Figure 6 – Estimate of factor 3

Figure 6 refers to the estimate of the factor f3, as in $s_t = \sum_{j=1}^{n} \delta_j f_{jt} + v_t + c$. This factor is standardized such that it has a mean of 0. The estimate follows the standard case for this study using 152 monthly samples and 19 exchange rates, and the factor model is estimated by maximum likelihood.



In summary, the analysis with respect to the interpretation of the factors is very subjective, and the factors might represent important unobservable variables like preferences, risk premium or investors' risk aversion. However, it is possible that the factors carry useful information with which to forecast the exchange rate. This analysis will be performed in the next section.

4.2 Forecasting Results

4.2.1 Full Sample

Initially, we test the forecasting predictions of the different models, including the factors with the random walk model. Table 1shows the results, considering the entire sample from January 1999 to August 2011.

Table 1 – Forecasting Results for the entire period

Table 1 shows the Theil's U statistics for the different models and forecasting horizons. The statistics are calculated using the random walk model without trends such as the *benchmark model* and comparing the forecast value of the models with factors with and without macroeconomic fundamentals. The U statistic is defined as the ratio between the root mean square prediction error and that given by the random walk model. Values below unity indicate that the model was superior to the random walk model. The complete sample corresponds to the period from January 1999 to August 2011 and consists of 152 monthly observations.

	Forecasting Horizon (h)			
Model	**1 mo**	**3 mo**	**6 mo**	**12 mo**
Only Factors (N=3)	1.036	1.087	1.291	1.283
Factors + Taylor	1.027	1.079	1.273	1.286
Factors + Monetary Model	1.075	1.147	1.480	1.392
Factors + PPP	1.026	1.056	1.203	1.245

The results in table 1 indicate that the Theil's U statistics exhibit values greater than 1 during the floating exchange regime in Brazil, which suggests that these models fail to give superior predictions to the random walk model with respect to the behavior of the Brazilian exchange rate. This result is shown to be valid independent of the forecast horizon. For the complete sample, there is no evidence that the models exhibit particularly different performance as a function of the forecast horizon. No one model in any forecasting horizon is able to beat the random walk.

4.2.2 Subsamples

To evaluate different periods, the complete sample was divided into 5 subsamples, as illustrated in figure 7 (in which the hatched area represents the period covered by the sample). In each subsample, the 4 specifications of the model undergo 16 simulations and are evaluated by their forecast for the 1-, 3-, 6- and 12-month horizons.

Figure 7 – Representation of the subsamples

Figure 7 illustrates the composition of each subsample. From 1999 to 2011, each year contains 12 data points, which correspond to the value of the last day of each month. The sample for 2011 contains 8 data points, covering the period from January to August.

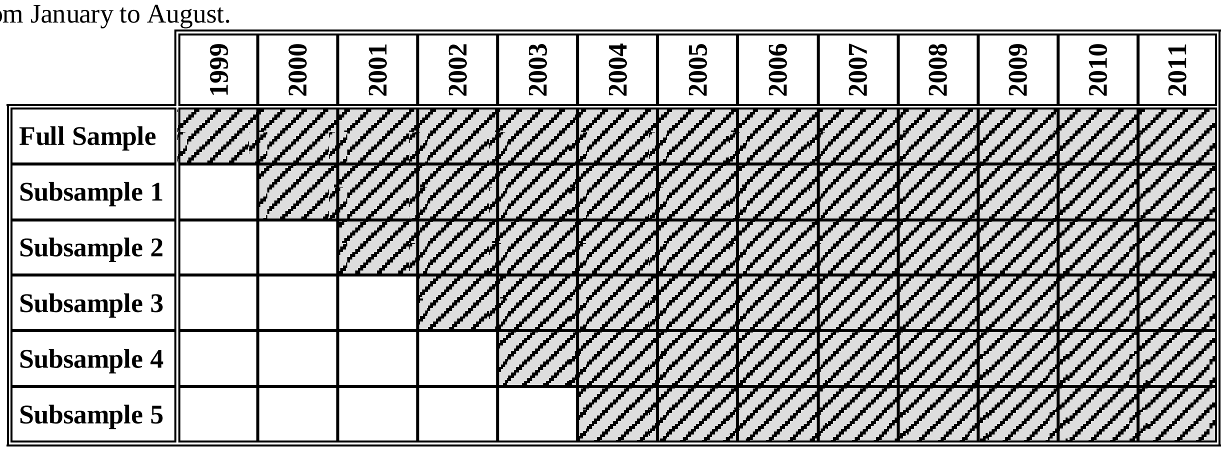


Table 2 shows the consolidated results. Although the results obtained for the complete sample suggest that the models do not possess smaller forecasting errors than the random walk model, analysis of the subsamples may suggest otherwise. If the sample of the initial floating exchange period in Brazil is ignored, then the forecast error decreases, and some models outperform the random walk in terms of forecasting errors.

Table 2 – Results for the Subsamples

Table 2 shows the results for the forecasting of the different models and forecasting horizons. The estimation period is the period in which the forecast model is estimated. The forecast period is the portion of the sample in which the model is compared to the random walk model under the root mean square prediction error criterion. The median Theil's U is given for each horizon and each sample by the median of the U statistics for the 4 specifications covered in this study. In the table, #U<1 describes how many of the four specifications had a U statistic below unity. This indicates that the model exhibited better out-of-sample performance than the random walk model. CW p-value<0.1 indicates the number of specifications for which the null hypothesis of the Clark and West (2006) test would be rejected at 10%.

				Forecasting Horizon (h)			
Sample	**Estimation Period**	**Forecasting Period**	**Statistic**	**1 mo**	**3 mo**	**6 mo**	**12 mo**
Full	From 01/99 To [08/10 - 07/11]	From 01/04 To 08/11	Median Theil's U #U<1 / CW p-value<0.10	1.041 0 / 0	1.092 0 / 0	1.312 0 /0	1.302 0 / 0
Subsample 1	From 01/00 to [08/10 - 07/11]	From 01/05 To 08/11	Median Theil's U #U<1 / CW p-value<0.10	1.039 0 / 0	1.081 0 / 0	1.298 0 / 0	1.261 0 / 0
Subsample 2	From 01/01 To [08/10 - 07/11]	From 01/06 To 08/11	Median Theil's U #U<1 / CW p-value<0.10	0.988 3 / 3	1.005 2 / 2	1.136 0 / 0	1.133 0 / 0
Subsample 3	From 01/02 To [08/10 - 07/11]	From 01/07 To 08/11	Median Theil's U #U<1 / CW p-value<0.10	0.994 2 / 2	1.004 2 / 2	1.119 0 / 0	1.129 0 / 0
Subsample 4	From 01/03 To [08/10 - 07/11]	From 01/08 To 08/11	Median Theil's U #U<1 / CW p-value<0.10	0.964 4 /4	0.969 4 / 2	1.041 1 / 1	1.001 2 / 2
Subsample 5	From 01/04 To [08/10 - 07/11]	From 01/09 To 08/11	Median Theil's U #U<1 / CW p-value<0.10	1.016 1 / 1	1.027 1 / 1	1.136 1 / 1	0.976 3 / 3

Starting in 2001 when the samples for the years 1999 and 2000 are excluded, 3 models are able to beat the random walk for 1-month horizon forecasting and 2 beat the random walk for a 3-month forecasting. Similar pattern arises when we use the subsample 3 with 2 models being superior the random walk for a short-horizon forecasting.

The results shown in table 2 are more promising for the use of the models in forecasting the exchange rate when we only consider data after 2003. The results indicate that at least one model beat the random walk for any forecasting horizon, and for subsample 4, for the 1-month prediction, all 4 models have superior performance to the random walk. Notably, there is an improvement in the medium-term forecasting results; before 2002, no model could beat the random walk for the 6 and 12-month predictions, but in subsamples 4 and 5, some models perform better than the random walk for these forecasting horizons.

Table 3 – Results by Model–Subsample 2004–2011

Table 3 shows the results by model for the period from January 2004 to August 2011. U statistics correspond to the ratio between the root mean square prediction error of the model and the data for the random walk model. The CW statistic was developed by Clark and West (2006) and is asymptotic and one-tailed. Under the null hypothesis, the root mean square prediction error for the model that includes factors and fundamentals does not differ from the root mean square prediction error given by the random walk model. *, **, *** indicate rejection of the null hypothesis at 10%, 5% and 1%, respectively.

		Forecast Horizon (h)			
Model	**Statistic**	**1 mo**	**3 mo**	**6 mo**	**12 mo**
Only Factors (N=3)	Theil's U	0.972*	0.99	1.079	1.019
	CW	1.58	1.16	-0.98	-0.07
Factors + Taylor	Theil's U	0.949**	0.952***	1.015	0.984***
	CW	1.89	2.62	0.60	3.48
Factors + Monetary Model	Theil's U	0.973*	0.986	1.134	1.06
	CW	1.40	1.06	-0.89	-1.55
Factors + PPP	Theil's U	0.961**	0.948**	0.936**	0.942***
	CW	1.82	2.34	2.05	2.47

The forecast statistics for each model considering subsample 4 are shown in Table 3.[5]Results confirm that for this subsample, the U statistics drop below unity. All models are able to beat the random walk for the 1-month forecasting horizon, and the model that includes PPP in its specification is superior to the random walk for all forecasting-horizons. These results are consistent with those obtained in Engel and West (2008), in which the model that includes the PPP generally exhibited the best forecast performance. The model that includes the factors and the Taylor rule generate lower forecasting errors than the random except for a 6-months forecasting horizon. Finally, the results in table 3 note that the model containing only the factors beats the random walk for only a short forecasting horizon (1 month).

4.3 Information contained in the factors

One of the objectives of the study was to determine whether it is justifiable to use unobservable factors as explanatory variables of the exchange rate and whether these factors contribute additional information that is not contained in the observable macroeconomic series. Table 4 shows Theil's U statistics for subsample 4, which excludes the initial period of the exchange rate, for specifications with and without factors.

[5] Only results for subsample 4 are shown here. This was done only to avoid the presentation of several tables in the text. Results for all subsamples are show in the appendix.

Table 4 – Theil's U statistics with added factors

Table 4 compares the forecasting of the models with and without the inclusion of the factors for the period from January 2004 to August 2011. U statistics correspond to the ratio between the root mean square prediction error for the model and the data from the random walk model. In the table, Δ corresponds to the variation between the U statistics before and after the addition of factors to the specification of the model with fundamentals.

	Taylor			Monetary Model			PPP		
Horizon (h)	**Without Factors**	**With Factors**	**Δ**	**Without Factors**	**With Factors**	**Δ**	**Without Factors**	**With Factors**	**Δ**
1 mo	1.042	0.949	-0.093	1.033	0.973	-0.060	1.044	0.961	-0.083
3 mo	1.126	0.952	-0.174	1.105	0.986	-0.119	1.145	0.948	-0.197
6 mo	1.218	1.015	-0.203	1.203	1.134	-0.069	1.283	0.936	-0.347
12 mo	1.371	0.984	-0.387	1.376	1.060	-0.316	1.456	0.942	-0.514

In all of the specifications, incorporating the factors reduced the root mean square prediction error, most notably in longer forecast horizons. This result suggests that by adding factors that capture latent variables in the exchange rates, it is possible to extract unobserved information from the series of observed economic fundamentals and to improve the forecasting power of the models. Importantly no one model would be able to beat the random walk in the period without the inclusion of the factors, a strong indication that not only do the factors carry some information with which to forecast the exchange rate but they also are fundamental to study the dynamics of the exchange rate.

5. Conclusions

This paper studies the usefulness of the use of factors embedded in the common movements of exchange rates in forecasting the exchange rate of the Brazilian Real/ US Dollar from January 1999 to August 2011.

The results indicate that when the forecasting exercise is performed using the full sample from January 1999 until 2011,no one model containing only the factors or with the factors together with macroeconomic variables is able to outperform the predictions given by a random walk. In this sense, the addition of factors to the traditional macroeconomic models does not seem to be fruitful to forecast the exchange rate.

In contrast with these results, when we perform similar forecasting exercises considering different subsamples, there is evidence that models including the factors alone or together with

macroeconomic variables outperformed the random walk model. This result is especially valid for subsamples that exclude the initial period of the floating exchange regime.
Furthermore, the results indicate that when the forecasting given by the models with and without the addition of the factors are compared, the models containing the factors are superior compared to the models containing only macroeconomic variables, confirming that the factors enclose useful information to forecast the exchange rate.

In summary, the results of the paper note that the use of factors is useful to forecast the exchange rate. There are periods when the unobservable variables captured by the estimation of the factors from the common dynamics of exchange rates are crucial for forecasting the exchange rate. However, the inclusion of the factors alone is not able to eliminate the instability of the relationship between the exchange rate and the fundamentals. Thus, different variables or approaches are necessary to model the behavior of the exchange rate.

One line of research is suggested and analyzed by Bachetta and Wincoop (2004, 2011), Sarno and Valente (2009) and Fratscher, Sarno and Zinna (2012). The scapegoat theory asserts that if the dynamics of the exchange rate is partially given by unobservable variables, as shown in this paper, changes in the expectations of the agents with respect to the structural parameters of the economy generated by shocks on these unobservable variables will generate this instability on the relationship between the exchange rate and fundamentals. One line of research could verify whether this theory improves the forecasting power of the models and build theoretical models to rationalize this behavior.

Another line of research suggested by Evans and Lyons (2002, 2005, 2008) and Chinn and Moore (2010) is to add a microstructure approach to the traditional macroeconomic models. In these papers, the inclusion of order flow variables would solve the problem of the conventional models because these variables would account for shocks that lead to instability in the relationship between the exchange rate and fundamentals.

REFERENCES

Bai, J. (2004). Estimating Cross-Section Common Stochastic Trends in Nonstationary Panel Data. *Journal of Econometrics*, 122, 137-183.

Bacchetta, P. & van Wincoop, E. (2004). A Scapegoat Model of Exchange Rate Determination. *American Economic Review, Papers and Proceedings,* 94, 114-18.

Bacchetta, P. & van Wincoop, E. (2006). Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle? *American Economic Review,* 96, 552-576.

Bacchetta, P. & van Wincoop, E. (2011). On the Unstable Relationship between Exchange Rates and Macroeconomic Fundamentals, Manuscript.

Cheung, Y., Chinn, M. & Pascual, A. (2005). Empirical Exchange Rate Models of the Nineties: Are Any Fit to Survive? *Journal of International Money and Finance,* 24, 1150-1175.

Cheung, Y., Chinn, M. & Pascual, A. (2005b). What Do We Know about Recent Exchange Rate Models? In-Sample Fit and Out-of-Sample Performance Evaluated. in Paul De Grauwe, ed., Exchange Rate Economics: What Do we Stand?, The MIT Press: Chapter 8, 239-76.

Chinn, M. & Meese, R. (1995). Banking on Currency Forecasts: How Predictable is Change in Money? *Journal of International Economics,* 38, 161–178.

Chinn, M. & Moore, M. (2010). Order Flow and the Monetary Model of Exchange Rates: Evidence from a Novel Data Set. Manuscript.

Clark, T. & West, D. (2006). Using Out-of-Sample Mean Squared Prediction Errors to Test the Martingale Difference Hypothesis. *Journal of Econometrics,* 135, 155-186.

Clark, T. & West, K. (2007). Approximately Normal Tests for Equal Predictive Accuracy in Nested Models. *Journal of Econometrics,* 138, 291-311.

Engel, C. & West, K. (2005). Exchange Rates and Fundamentals. *Journal of Political Economy,* 113, 485-517.

Engel, C., Mark, N. & West, K. (2007). Exchange Rate Models Are Not As Bad As You Think., in NBER Macroeconomics Annual, 2007, D. Acemoglu, K. Rogoff and M. Woodford (eds.), Chicago: University of Chicago Press.

Engel, C., Mark, N. & West, K. D. (2008). Factor Model Forecasts of Exchange Rates, Working Paper, Manuscript.

Engel, C. & West, K. (2006). Taylor Rules and the Deutschmark-Dollar Real Exchange Rate. *Journal of Money, Credit and Banking*, 38, 1175-1194.

Evans, M. & Lyons, R. (2002). Order Flow and Exchange Rate Dynamics. *Journal of Political Economy*, 110, 170–180.

Evans, M. & Lyons, R. (2005). Meese-Rogoff Redux: Micro-based Exchange Rate Forecasting, *American Economic Review* Papers and Proceedings, May, 405-414.

Evans, M. & Lyons, R. (2008). How Is Macro News Transmitted to Exchange Rates?.*Journal of Financial Economics,* 88, 26-50.

Faust, J., Rogers J.H. & Wright J.H. (2003). Exchange rate forecasting: the errors we've really made. *Journal of International Economics*, 60, 35-59.

Forni, M. & Reichlin, L. (1998). Let's Get Real: A Factor Analytic Approach to Disaggregated Business Cycle Dynamics. *Review of Economic Studies,* 65, 453-473.

Forni, M., Hallin, M., Lippi, M. & Reichlin, L. (2000). The Generalized Dynamic Factor Model: Identification and Estimation. *Review of Economics and Statistics,* 82, 540-554.

Fratzscher, M., Sarno, L.; & Zinna, G. (2012). The scapegoat theory of exchange rates: The first tests. European Central Bank Working Paper 1418.

Groen, J.J. (2005). Exchange Rate Predictability and Monetary Fundamentals in a Small Multi-Country Panel. *Journal of Money, Credit and Banking,* 37, 495-516.

Groen, J. J. (2006). Fundamentals Based Exchange Rate Prediction Revisited. Manuscript, Bank of England.

Mark, N. (1995). Exchange Rates and Fundamentals: Evidence on Long-Horizon Predictability. *American Economic Review,* 85, 201-218.

Mark, N. (2008). Changing Monetary Policy Rules, Learning and Real Exchange Rate Dynamics. Manuscript, University of Notre Dame.

Mark, N. & Donggyu, S. (2001). Nominal Exchange Rates and Monetary Fundamentals: Evidence from a Small Post-Bretton Woods Sample. *Journal of International Economics* 53, 29-52.

Meese, R. & Rogoff, K. (1983). Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample? *Journal of International Economics,* 14, 3-24.

Moura, M., Lima, A. & Mendonça, R. (2008). Exchange Rate and Fundamentals: The Case of Brazil. *Revista de Economia Aplicada,* 12, 395-416.

Muinhos, M., Alves, S. & Riella, G. (2003). Modelo macroeconômico com setor externo: endogeneização do prêmio de risco e do câmbio. *Pesquisa e Planejamento Econômico,* 33, 61-89.

Sarno, L. & Valente, G. (2009). Exchange Rates and Fundamentals: Footloose or Evolving Relationship? *Journal of the European Economic Association*, 7, 786-830.

Sarno, L. & Taylor, M. (2002). The Economics of Exchange Rates. Cambridge and New York: Cambridge University Press.

Stock, J. H. & Watson, M. (2002). Forecasting Using Principal Components from a Large Number of Predictors. *Journal of the American Statistical Association,* 97, 1167-1179.

APPENDIX

Table A.1- Data Description

Table A.1 shows the data used throughout the text. All of the exchange rates used are from countries that have independent monetary policies and a floating exchange rate, according to the International Monetary Fund (IMF) classification: http://www.imf.org/external/np/mfd/er/2008/eng/0408.htm.

Variable	Country	Description	Source
Exchange Rate	Switzerland	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	Norway	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	Euro Zone	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	New Zealand	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	United Kingdom	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	Sweden	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	Japan	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	Poland	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	Canada	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	Australia	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	Iceland	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	South Korea	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	South Africa	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	Israel	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	Mexico	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	Chile	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	Philippines	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	Turkey	log (Domestic Currency / Foreign Currency)	Bloomberg
Exchange Rate	Brazil	log (Domestic Currency / Foreign Currency)	Bloomberg
Industrial Production	Brazil	Index Jan 2002=100 with seasonal adjustment	IBGE
Industrial Production	United States	Index Jan 2007=100 with seasonal adjustment	Federal Reserve
Prices	Brazil	IPCA -Index Dec 98=100	IBGE
Prices	United States	CPI - Index Dec 82=100	BLS
Money Supply	Brazil	R$ millions with seasonal adjustment	Central Bank
Money Supply	United States	US$ billions with seasonal adjustment	Federal Reserve

Table A.2 – Cointegration Tests

Table A.2 shows the statistics for the cointegration test described by MacKinnon (1990). Asymptotic critical values are assumed for a statistic with no trend corresponding to $\Delta\hat{\varepsilon}_t = \gamma\hat{\varepsilon}_{t-1} + u_t$

Model	t-Statistics	P-Value
Only Factors (N=3)	-4.81	0.07%
Factors + Taylor	-4.88	0.05%
Factors + Monetary Model	-6.07	0.00%
Factors + PPP	-5.12	0.02%

Table A.3 – Description of sub-samples

Table A.3 shows the division of the sample in different periods. For a specific date, the factors and explanatory variables are the same, independent of the forecast horizon. Estimation of the forecast model varies with *h*. The sample for estimation decreases as the horizon increases.

Sample	Estimation	Forecast	Sample Size	Horizon (h)
Full Sample	1999:1 - (2010:7+h)	(2003:12+h) - 2011:8	152	1,3,6 e 12 months
Subsample 1	2000:1 - (2010:7+h)	(2004:12+h) - 2011:8	140	1,3,6 e 12 months
Subsample 2	2001:1 - (2010:7+h)	(2005:12+h) - 2011:8	128	1,3,6 e 12 months
Subsample 3	2002:1 - (2010:7+h)	(2006:12+h) - 2011:8	116	1,3,6 e 12 months
Subsample 4	2003:1 - (2010:7+h)	(2007:12+h) - 2011:8	104	1,3,6 e 12 months
Subsample 5	2004:1 - (2010:7+h)	(2008:12+h) - 2011:8	92	1,3,6 e 12 months

Table A.4 – Loadings from estimation of the factors

Table A.4 shows the loadings estimated for the different exchange rates and factors. The loadings are estimated by the maximum likelihood with non-rotated factors using the complete sample (152 monthly data and 19 exchange rates). Loadings refer to the factor model, as $\mathrm{ins}_t = \sum_{j=1}^{n} \delta_j f_{jt} + v_t + c$, for the case of *n* = 3.

Country	$\hat{\delta}_1$	$\hat{\delta}_2$	$\hat{\delta}_3$
Switzerland	0.95	-0.19	-0.20
Norway	0.97	0.10	-0.14
Euro Zone	0.96	0.05	-0.13
New Zealand	0.96	0.18	-0.08
United Kingdom	0.62	0.73	0.01
Sweden	0.96	0.20	0.02
Japan	0.66	-0.57	0.02
Poland	0.90	0.02	0.04
Canada	0.96	-0.08	0.05
Australia	0.98	-0.04	0.05
Iceland	-0.06	0.97	0.17
South Korea	0.56	0.63	0.27
South Africa	0.37	0.46	0.39
Israel	0.61	-0.50	0.41
Mexico	-0.67	0.36	0.42
Chile	0.66	-0.06	0.69
Philippines	0.23	-0.30	0.75
Turkey	-0.36	0.13	0.77
Brazil	0.47	-0.36	0.78

Table A.5– U and CW statistics for (i) specification, (ii) horizon and (iii) subsample

Table A.5 presents the forecasting statistics for different models and horizons. The statistics were calculated using a random walk model with no trend as a benchmark model and by comparing the value forecast by the models with fundamentals and factors with this model. The RMSPE ratio is defined as the ratio between the root mean square prediction error of the forecast of the model and the error given by the random walk model. Values below unity indicate that the model exhibited better out-of-sample performance than the random walk model. The test statistic presented was developed by Clark and West (2006) and is asymptotic and one-tailed. Under the null hypothesis, the root mean square prediction error of the model that includes economic factors and macroeconomic fundamentals does not differ from the root mean square prediction error given by the random walk model.

Full Sample	**Statistic**	**Horizon (h)**			
Model		**1 mo**	**3 mo**	**6 mo**	**12 mo**
Only Factors (N=3)	RMSPE Ratio	1.036	1.087	1.291	1.282
	Statistic	0.570	-0.670	-2.160	-1.800
	P-Value	0.280	0.750	0.980	0.960
Factors+Taylor	RMSPE Ratio	1.027	1.079	1.273	1.286
	Statistic	0.960	0.080	-1.540	-1.690
	P-Value	0.170	0.470	0.940	0.950
Factors + Monetary Model	RMSPE Ratio	1.075	1.147	1.480	1.392
	Statistic	0.210	-0.940	-2.260	-1.760
	P-Value	0.420	0.830	0.990	0.960
Factors + PPP	RMSPE Ratio	1.026	1.056	1.203	1.245
	Statistic	0.900	0.140	-0.600	-1.530
	P-Value	0.180	0.440	0.730	0.940

Subsample 3	**Statistic**	**Horizon (h)**			
Model		**1 mo**	**3 mo**	**6 mo**	**12 mo**
Only Factors (N=3)	RMSPE Ratio	1.000	1.021	1.143	1.132
	Statistic	1.090	0.320	-1.910	-1.280
	P-Value	0.140	0.380	0.970	0.900
Factors+Taylor	RMSPE Ratio	0.978	0.990	1.085	1.093
	Statistic	1.550	1.640	-0.380	-0.940
	P-Value	0.060	0.050	0.650	0.830
Factors + Monetary Model	RMSPE Ratio	1.013	1.028	1.234	1.220
	Statistic	0.800	0.250	-1.790	-1.280
	P-Value	0.210	0.400	0.960	0.900
Factors + PPP	RMSPE Ratio	0.983	0.977	1.015	1.072
	Statistic	1.510	1.620	1.100	-0.590
	P-Value	0.070	0.050	0.140	0.720

Subsample 1	**Statistic**	**Horizon (h)**			
Model		**1 mo**	**3 mo**	**6 mo**	**12 mo**
Only Factors (N=3)	RMSPE Ratio	1.038	1.083	1.289	1.244
	Statistic	0.560	-0.510	-2.140	-1.770
	P-Value	0.290	0.700	0.980	0.960
Factors+Taylor	RMSPE Ratio	1.024	1.064	1.250	1.221
	Statistic	0.990	0.350	-1.370	-1.720
	P-Value	0.160	0.360	0.920	0.960
Factors + Monetary Model	RMSPE Ratio	1.069	1.131	1.459	1.380
	Statistic	0.260	-0.720	-2.120	-1.690
	P-Value	0.400	0.770	0.980	0.950
Factors + PPP	RMSPE Ratio	1.025	1.047	1.193	1.198
	Statistic	0.920	0.350	-0.430	-1.480
	P-Value	0.180	0.360	0.670	0.930

Subsample 4	**Statistic**	**Horizon (h)**			
Model		**1 mo**	**3 mo**	**6 mo**	**12 mo**
Only Factors (N=3)	RMSPE Ratio	0.972	0.990	1.079	1.019
	Statistic	1.580	1.160	-0.980	-0.070
	P-Value	0.060	0.120	0.840	0.530
Factors+Taylor	RMSPE Ratio	0.949	0.952	1.015	0.984
	Statistic	1.890	2.620	0.600	3.480
	P-Value	0.030	0.000	0.270	0.000
Factors + Monetary Model	RMSPE Ratio	0.973	0.986	1.134	1.060
	Statistic	1.400	1.060	-0.890	-1.550
	P-Value	0.080	0.150	0.810	0.940
Factors + PPP	RMSPE Ratio	0.961	0.948	0.936	0.942
	Statistic	1.820	2.340	2.050	2.470
	P-Value	0.030	0.010	0.020	0.010

Subsample 2	**Statistic**	**Horizon (h)**			
Model		**1 mo**	**3 mo**	**6 mo**	**12 mo**
Only Factors (N=3)	RMSPE Ratio	0.993	1.021	1.158	1.132
	Statistic	1.420	0.470	-1.490	-0.870
	P-Value	0.080	0.320	0.930	0.810
Factors+Taylor	RMSPE Ratio	0.976	0.992	1.099	1.094
	Statistic	1.780	1.620	-0.320	-0.650
	P-Value	0.040	0.050	0.620	0.740
Factors + Monetary Model	RMSPE Ratio	1.005	1.031	1.257	1.234
	Statistic	1.170	0.390	-1.520	-0.980
	P-Value	0.120	0.350	0.940	0.840
Factors + PPP	RMSPE Ratio	0.977	0.976	1.028	1.073
	Statistic	1.820	1.660	1.000	-0.410
	P-Value	0.030	0.050	0.160	0.660

Subsample 5	**Statistic**	**Horizon (h)**			
Model		**1 mo**	**3 mo**	**6 mo**	**12 mo**
Only Factors (N=3)	RMSPE Ratio	1.019	1.043	1.149	0.955
	Statistic	0.720	0.160	-1.630	3.010
	P-Value	0.240	0.440	0.950	0.000
Factors+Taylor	RMSPE Ratio	1.019	1.013	1.077	0.944
	Statistic	0.950	0.890	-0.020	3.990
	P-Value	0.170	0.190	0.510	0.000
Factors + Monetary Model	RMSPE Ratio	1.035	1.081	1.310	1.093
	Statistic	0.040	-0.240	-2.340	-2.310
	P-Value	0.480	0.600	0.990	0.990
Factors + PPP	RMSPE Ratio	0.989	0.969	1.006	0.912
	Statistic	1.390	1.350	1.610	2.490
	P-Value	0.080	0.090	0.050	0.010