Insper
Programa de Mestrado Profissional em Economia

Débora Darin

Carry and momentum in the Brazilian yield curve

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Dissertação apresentada ao Programa de Mestrado Profissional em Economia do Insper como parte dos requisitos para obtenção do título de Mestre em Economia.

Área de concentração: Economia dos Negócios

Linha de pesquisa: Finanças

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ABSTRACT

The return premia of the yield curve can be comprehensively described by a perspective of style factors, a set of characteristics that includes measures such as value, momentum and carry, applied to bonds, according to Brooks and Moskowitz (2017). Based on the referred article, this paper focuses on the information about expected returns that can be extracted from the slope and the curvature of the yield curve (therefore, carry and momentum strategies) across all levels of the yield curve, in a multi-maturity model. This study combines carry and momentum strategies applied to the Brazilian yield curve, and verifies whether it is possible to achieve superior returns by applying this combined style factor trading strategy to the curve, regardless of the interest rates levels in the time-series. Additionally, this study provides evidence on the application of carry and momentum strategies to the Brazilian One-day Interbank Deposit Futures, and verifies its adherence to the existing literature.

Keywords: Risk Premia. Yield Curve. Carry. Momentum.

RESUMO

O prêmio presente na curva de juros pode ser descrito de maneira abrangente através de uma perspectiva de *style factors*, um conjunto de características que inclui medidas como *value*, *momentum* e *carry* aplicadas a títulos de renda fixa, de acordo com Brooks e Moskowitz (2017). Com base no artigo citado, o proposto estudo discorre sobre os retornos que podem ser extraídos a partir da inclinação e da curvatura da curva de juros, em um modelo que explora e compara contratos com diferentes vencimentos. O estudo combina estratégias de *momentum* e *carry* ao longo da extensão da curva de juros brasileira, e verifica se é possível obter retornos em excesso à taxa de juros livre de risco ao aplicar uma estratégia combinada de fatores sistemáticos à curva, independentemente do nível das taxas de juros e de outras variáveis econômicas na série temporal. Adicionalmente, este estudo fornece evidências sobre a aplicação de estratégias de carry e momentum para os contratos futuros de DI (Futuro de Taxa Média de Depósitos Interfinanceiros de Um Dia), e verifica a aderência de seus resultados à literatura existente.

Palavras-chave: Prêmio de Risco. Curva de Juros. Carry. Momentum.

EXECUTIVE SUMMARY

In the financial market, traders benefit from finding and exploring price asymmetries. As explored in the financial literature, Brooks and Moskowitz (2017) associate the yield curve premia with characteristics that include measures such as value, momentum and carry, a pattern that has been similarly found in the Brazilian yield curve by Shousha (2005).

This study extends the present literature, applying concepts of the yield curve premia to the Brazilian yield market in order to quantify its results through a set of performance measures.

Through developing a quantitative trading model, this paper focuses on analyzing the premium extraction from the Brazilian yield curve. The analysis will begin from extracting information about expected returns from the slope and curvature of the curve across its different levels (multi-maturity model), verifying if it is possible to achieve consistent returns from these factors, and then proceeding to compare the achieved performances between contracts with different maturities, not only within the same strategy, but also between different strategies.

Much of the knowledge regarding trading strategies often keeps restrained to players in the financial markets industry. Therefore, this field has not been completely explored in the academic scope. The findings in this study should enrich the knowledge on both the fields of trading strategies and of the Brazilian interest rates market, and consequently open opportunities for further academic development in each of these areas.

The model focuses on the premium in the Brazilian yield curve that can be extracted from alternative factors that are purely technical, and do not directly rely on monetary policy decisions or economic forecasts. Therefore, the model elaborates on a Carry Strategy, a Momentum Strategy, and a set of combinations of both models, in order to result in a superior strategy. The combinations tested variations with and without risk control, and three variations of trading frequency: daily, weekly and monthly.

An asset's carry can be defined as its futures (or synthetic futures if none exist) return, assuming that prices stay the same, as defined by Koijen et al. (2018). It is a model-free characteristic that is directly observable ex-ante from futures prices, whereas the expected price appreciation must be estimated using an asset pricing model.

The concept of time-series momentum is explored by Moskowitz, Ooi and Pedersen (2012). Through this perspective, a security presents some persistence in returns for one to 12 months that partially reverses over longer horizons, consistent with sentiment theories of initial under-reaction and delayed over-reaction.

The analyzed assets were the Brazilian interest rate futures denominated "DIs", and traded on the Brazilian exchange "B3". The time series comprised data from ten years of observations, from eight different length vertices from the Brazilian yield curve.

When compared to the Brazilian risk-free rate (CDI), and to a pure carry and a pure momentum-based strategy, the combined strategies model proposed in this paper has proven to overperform not only the considered benchmark, but also both of the individual strategies. This exceeding performance has been further enhanced by the introduction of a risk management tool in the model, named a volatility cap.

Additionally, the paper results indicated that the Low Volatility Anomaly, a widely know phenomenon in the financial literature, can be observed among the Brazilian DI contracts, in line with Baker, Bradley and Wurgler (2011) findings. The contracts with shorter maturities, and shorter volatilities, have presented greater performances in comparison with the contracts with longer maturities.

The present research contributed to the field of quantitative finance as it aimed to verify if superior returns can be achieved through a combination of carry and momentum strategies, which are strategies based on the expected returns from the slope and curvature of the yield curve, in comparison to the application of each strategy separately or to the benchmark risk-free rate.

Moreover, this paper provided information about the application of carry and momentum strategies over the Brazilian yield curve, by means of a time series of exchange traded future contracts, in line with the theory found in previous studies from the financial literature that mainly focus on developed countries.

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1 INTRODUCTION

The return premia of the yield curve can be comprehensively described by a perspective of style factors, a set of characteristics that includes measures, such as value, momentum and carry, applied to bonds, according to Brooks and Moskowitz (2017).

This combination of characteristics comprises pricing information of the yield curve's three principal components (PCs), which are level, slope and curvature, as well as other important priced factors, such as macroeconomic variables (growth and inflation), and the Cochrane and Piazzesi (2005) factor, which predicts excess returns on one- to five-year maturity bonds based on a single tent-shaped linear combination of forward rates, as explored in Brooks and Moskowitz (2017).

In Brooks and Moskowitz (2017), value corresponds to the yield on the bond minus (maturity-matched) expected inflation ("real bond yield") and provides information about the level of yields in relation to a fundamental anchor – expected inflation; momentum provides information about recent trends in yield changes; and carry provides information about expected future yields assuming the yield curve stays the same. Therefore, value subsumes a "level" factor, while momentum and carry subsume information about expected returns from the slope and curvature of the yield curve, respectively. These characteristics describe both the cross-section and time-series of yield curve premia and connect to return predictability in other asset classes, suggesting a unifying asset pricing framework.

A similar pattern has been verified in the Brazilian yield curve by Shousha (2005), in a study that decomposes the variation in bond yields into six principal components, highlighting the components which appear to relate to the level and the slope of the yield curve.

An asset's carry can be defined as its futures (or synthetic futures if none exist) return, assuming that prices stay the same, as defined by Koijen et al. (2018). It is a model-free characteristic that is directly observable ex-ante from futures prices, whereas the expected price appreciation must be estimated using an asset pricing model. The concept of time-series momentum is explored by Moskowitz, Ooi and Pedersen (2012). Through this perspective, a security presents some persistence in returns for one to 12 months that partially reverses over longer horizons, consistent with sentiment theories of initial under-reaction and delayed over-reaction.

Based on the article of Brooks and Moskowitz (2017), and beyond the asset pricing literature, this paper focuses on the field of applied quantitative finance, applying concepts of the foreign literature to the Brazilian yield curve. Through calculating and analysing momentum signals, and combining them with a carry trading strategy, the present study aims to verify whether it is possible to achieve consistent returns in excess of the risk free rate, at different levels of the interest rates. The concept of value was not considered in this model, in order to produce a strategy that runs independently of economic variables such as the expected inflation, a variable that would have to be input from proprietary economic forecasts, or from sources such as the weekly "Focus Survey", published by the Brazilian Central Bank. Therefore, the model in this article will extract information about expected returns from the slope and curvature of the yield curve across all levels of the yield curve (multi-maturity model).

Additionally, this study also analyzes the outcome of both the carry and momentum strategies individually applied to the Brazilian market, and verifies its adherence to the existing literature through outcomes which should correspond to the theory presented by Moskowitz, Ooi and Pedersen (2012), Koijen et al. (2018) and Baz et al. (2015), which cover other markets and financial assets.

The main objective sought is to design a more comprehensive approach for the yield curve premium. The proposed model, combining both carry and momentum strategies, makes possible the identification of yield premia for several different maturities across the Brazilian yield curve, each of them represented by a one-day interbank deposit future contract traded in Brazil.

The analysed premium should be obtained through the slope and curvature of the yield curve or, in other words, this study focuses on the premia that can be obtained at every level of yield in the market, across all different maturities along the yield curve. This characteristic should be verified after backtesting the carry and momentum model. By combining carry and momentum strategies across the curve, the ultimate goal is to generate a trading strategy that is capable of extracting the most from interest rates premium across different maturities, regardless of the level of the yields negotiated in the country.

2 LITERATURE REVIEW

2.1 Term Structure of Interest Rates

This section describes the yield curve and its components, as this study's models, based on style factors, were performed within the Brazilian yield curve futures market.

An interest rate in a particular situation defines the amount of money a borrower promises to pay the lender, as described by Hull et al. (2009). Moreover, Hull et al. (2009) defines the n-year zero-coupon interest rate as the rate of interest earned on an investment that starts today and lasts for n years, with all the interest and principal being realized at the end of n years (there are no intermediate payments).

Assets of different maturities typically sell at different yields to maturity. When these asset prices and yields are compiled, long-term assets sell at higher yields than short-term assets. Bodie et al. (2013) summarize the relationship between yield and maturity graphically in a yield curve, which is a plot of yield to maturity as a function of time to maturity.

Forward interest rates are the future rates of interest implied by current zero rates for periods of time in the future, according to Hull et al. (2009). Caldeira, Moura and Portugal (2010) claim that the term structure of interest rates is represented by a set spot rates for different maturities. A number of different theories have been proposed in order to explain what determines the shape of the zero curve, as described by Hull et al. (2009).

The expectations theory conjectures that long-term interest rates should reflect expected future short-term interest rates; more precisely, it argues that a forward interest rate corresponding to a certain future period is equal to the expected future zero interest rate for that period.

Another idea, market segmentation theory, conjectures that there need be no relationship between short-, medium-, and long-term interest rates. Under the theory, a major investor such as a large pension fund or an insurance company invests in bonds of a certain maturity and does not readily switch from one maturity to another. The short-term interest rate is determined by supply and demand in the short-term bond market; the medium-term interest rate is determined by supply and demand in the medium-term bond market; and so on.

The theory that is most appealing is liquidity preference theory. The basic assumption underlying the theory is that investors prefer to preserve their liquidity and invest funds for short periods of time. Borrowers, on the other hand, usually prefer to borrow at fixed rates for long periods of time. This leads to a situation in which forward rates are greater than expected future zero rates. The theory is also consistent with the empirical result that yield curves tend to be upward sloping more often than they are downward sloping.

In fixed income, the term structure is, more often than not, upward sloping. Conventional explanations range from the liquidity theory of rates - investors should be compensated for the higher risk of holding long bonds, hence the upward sloping yield curve - to the theory of preferred habitats - investors prefer short bonds to long bonds. Rates tend to roll down the curve: this translates into excess returns for fixed income investors, as described by Baz et al. (2015).

At any point of time t, there will be a collection of zero-coupon bonds that differ only in terms of maturity. However, in a given moment, there may not be a bond available to all desired maturities as bonds are not negotiated for all possible maturities, as explain Caldeira, Moura and Portugal (2010).

The interpolation method development can be found in McCulloch (1971) and McCulloch (1975). The fundamental idea behind cubic spline interpolation is based on the engineer's tool used to draw smooth curves through a number of points. This spline consists of weights attached to a flat surface at the points to be connected according to McKinley and Levine (1998). A flexible strip is then bent across each of these weights, resulting in a pleasingly smooth curve.

Real world numerical data is usually difficult to analyze. Any function which would effectively correlate the data would be difficult to obtain and highly unwieldy. To this end, the idea of the cubic spline was developed. Using this process, a series of unique cubic polynomials are fitted between each of the data points, with the stipulation that the curve obtained be continuous and appear smooth. As stated by McKinley and Levine (1998), these cubic splines can then be used to determine rates of change and cumulative change over an interval.

2.1.1 Brazilian Interest Rate Futures

The Brazilian interest rate futures market shows some particularities that distinguish it from most markets worldwide, as noted by Santos and Silva (2015).

In Brazil, the contract traded represents the present value of a virtual bond whose value at expiry is 100,000 BRL. Each DI contract has, as its underlying asset, the DI interest rate compounded until the contract's expiration date, for this purpose defined as the capitalized daily DI rates verified in the period between the trade date and the last trading day, as published by B3 (2020).

2.2 Alternative Risk Premia

According to Hamdan et al. (2016), alternative risk premia (ARP) are systematic risk factors that can help to explain the past returns of diversified portfolios, and it is also designated as non-traditional risk premia by Raiteri and Malagoli (2020). They may be risk premia in a strict sense, but also market anomalies or common strategies.

A style, as in Israel and Maloney (2014), is an alternative source of return which consists on a disciplined, systematic method of investing that can produce long-term positive returns across markets and asset groups, backed by robust data and economic theory.

2.2.1 Carry

Carry is the tendency for higher-yielding assets to provide higher returns than lower-yielding assets. (ISRAEL; MALONEY, 2014)

Koijen et al. (2018) define the carry of an asset as the return on a futures position, assuming its price stays constant; hence, carry is the forward-looking dividend yield in excess of r^f. Baz et al. (2015) define carry as the difference between the spot the spot and the forward price of an asset or, in other words, the profit in a long forward position if prices do not change.

According to Koijen et al. (2018) the concept has been mostly studied in the currency markets, but it can be applied to any asset. Any security or derivative expected return can be decomposed into its “carry”, an ex-ante and model-free characteristic, and its expected price appreciation. So, "carry" can be understood as the expected return of a security or derivative if there is no change in underlying prices; that is, if prices do not move and only time passes, that security or derivative will earn its "carry".

In line with Israel and Maloney (2014), the economic intuition behind carry is that it balances out supply and demand for capital across markets. High interest rates can signal an excess demand for capital not met by local savings; low rates suggest an excess supply. As traditional economic theory would argue, in the case of currencies, these rate differentials should be offset such that investor returns would be the same across markets, but the evidence shows otherwise. This may be due to the presence of non-profit-seeking market participants, such as central banks and corporate hedgers, introducing inefficiencies to currency markets and interest rates (ISRAEL; MALONEY, 2014).

The concept of carry has been shown to be a good predictor of returns cross-sectionally (going long securities with high carry while at the same time going short securities with low carry) and in time series (going long a particular security when carry is positive or historically high and short when carry is negative or historically low). So, the concept of "carry" provides a unifying framework for return predictability and gives origin to carry strategies across a host of different asset classes, including global equities, global bonds, commodities, US Treasuries, credit, and options. Carry strategies are commonly exposed to global recession, liquidity, and volatility risks, though none fully explain carry’s premium.

Koijen et al. (2018) is a great reference for discussing cross-sectional carry strategies in several different markets. There are typically market-neutral carry strategies where we are always long some currencies, rates, commodities and indices and short some others. Baz et al. (2015) discusses in detail the differences between cross-sectional strategies and time series strategies for three different factors, including carry. Baltas (2017) also looks at carry strategies, both cross-sectional and time series, across different futures markets (commodities, equity indices and government bonds), and discusses the benefits of constructing a multi-asset carry strategies

2.2.1.1 Carry Strategy Algorithm

The carry premium in this model corresponds to the performance accrued by holding each of the DI futures contracts, with different maturities. Therefore, the backtest corresponds to a long-only strategy with DI futures contracts: long UP, short yield.

In the strategy algorithm, the DI future contract is held from the initial trade date until 3 days before its expiry, being then rolled to the next available contract. The carry trade performance accounts for the premium accrual for holding future contracts.

2.2.2 Time Series Momentum

Trend-following is one of the most prevalent quantitative strategies and it applies to several markets such as equity indices, currencies, and futures markets such as commodities and bond futures. Trend-following (TF), or Time Series Momentum (TSM), refers to the predictability of the past returns on future returns and is the focus of several influential studies. In the academic summary of trend-following, Moskowitz, Ooi and Pedersen (2012) document significant momentum in monthly returns. There they find strongest results for relatively short holdings periods (1 or 3 months) and mid-range lookback periods such as (9 and 12 months).

Momentum consists on the tendency for an asset's recent relative performance to continue in the near future, as defined by Israel and Maloney (2014).

A univariate time-series momentum strategy is defined as the trading strategy that takes a long or short position in a single asset based on the sign of the recent asset return over a particular lookback period (BALTAS; KOSOWSKI, 2013). The classic definition of momentum is simply the percentage change in the financial time series over h time periods. Sometimes, this classic definition is calculated by looking at log-percentage changes by calculating the the difference between the log of today's price vs. the log of the price h periods ago.

In this model, the momentum metric $Mom_i(h)$ is associated with the concept of moving averages convergence and divergence, in consonance with Appel (2003).

2.2.2.1 Moving Average Convergence-Divergence

The Cross-Over Moving Average (or sometimes called MACD, for Moving Average Convergence-Divergence) is essentially a comparison between a short and a long moving average. It consists of a technical analysis tool that can be used to determine future trends in the stock market by using actual stock market activity to establish patterns of strength and weakness, as proposed by Appel (2003).

If the fast moving average is above the slow one, it means that the most recent information is pulling the price upward, signaling a positive momentum.

Although the MACD signal can be constructed with traditional moving averages of different periods, the most commonly used measure is the exponentially weighted moving average (EWMA). So the MACD indicator is given by:

$$MACD_{i,t}\left(m_1, m_2, \alpha\right) = EWMA_{i,t}\left(m_1, \alpha\right) - EWMA_{i,t}\left(m_2, \alpha\right) \tag{2.1}$$

where $m_1 < m_2$. The formula for the exponentially weighted moving average is:

$$EWMA_{i,t}\left(m, \alpha\right) = \frac{\sum_{l=1}^{m} \alpha^l ER_{i,t-l+1}}{\sum_{l=1}^{m} \alpha^l} \tag{2.2}$$

where m can take integer values from 1 to ∞, and $0 < \alpha < 1$ measures the speed of decay of the weights. The most common way to set the value of α is based on the half-life λ, which is interpreted as the number of periods it takes for the weight to reach 0.5.

$$\alpha = 1 - e^{-\frac{\ln(2)}{\lambda}}. \tag{2.3}$$

To illustrate the concept, in figure 1, the moving average convergence-divergence for the DIs with maturities of 3 months have been plotted. The short average and the long average have been calculated through the EWMA method, with half lives of 12 and 26 periods, respectively. These are the most common parameters, being recommended by Appel (2003).

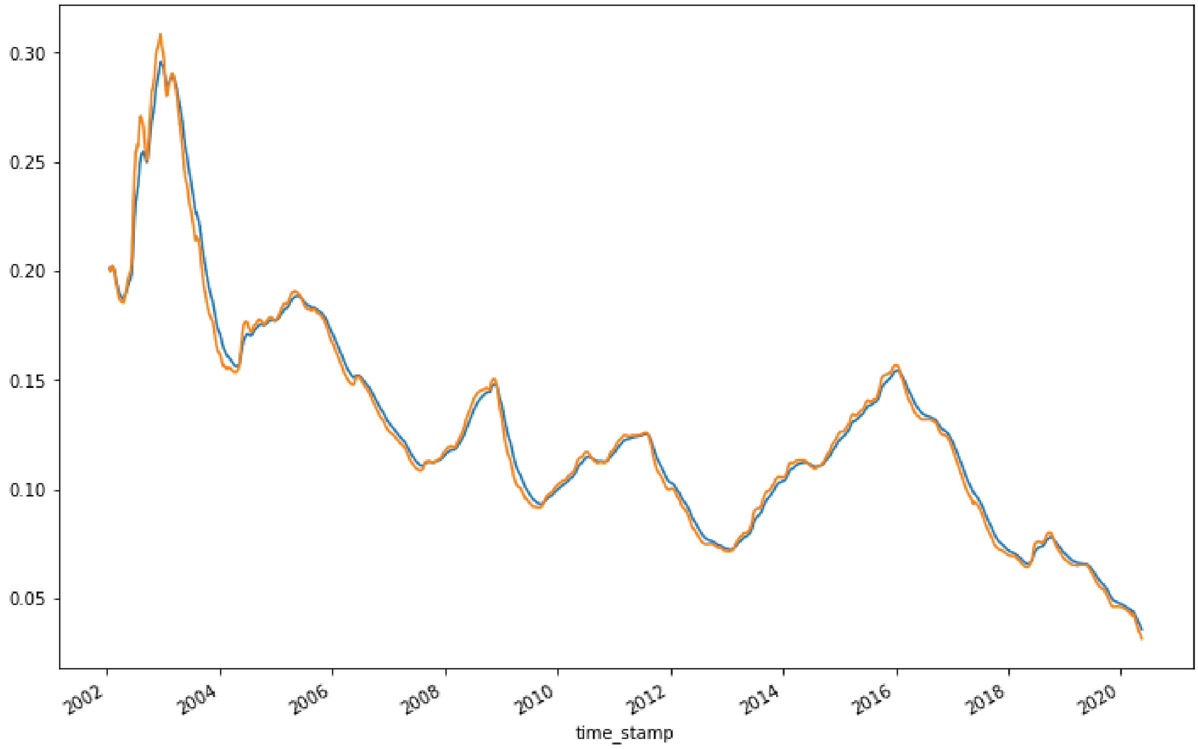

Figure 1 – MACD - 1 Year Contracts (EWMA half lives = 12 and 26 periods)

2.2.2.2 Momentum Algorithm

In this study, the momentum signal is based on the cross over of two exponentially weighted moving averages, in accordance with Appel (2003). The algorithm building that signal is the following:

1. Select the set of time-scales, consisting of a short and a long exponentially weighted moving average (EWMA);
2. In line with Appel (2003), the considered time-scales are $S_k = (12)$ and $L_k = (26)$. As explained by Baz et al. (2015), those numbers are not look-back days or half-life numbers. In fact, each number (n) translates to a lambda decay factor (λ) of $\frac{n}{n-1}$ to plug into the standard definition of an EWMA. The half-life (HL) is then given by:

$$HL = \frac{log(0.5)}{log(\lambda)} = \frac{log(0.5)}{log(1 - \frac{1}{n})} \tag{2.4}$$

3. Interpolate the yield curve, in order to obtain the respective yield for each contract and date in the time series;
4. For each contract and date, calculate both averages of the corresponding yields: $EWMA[P|S_k]$ (short period, or "fast" moving average) and $EWMA[P|L_k]$ (long period, or "slow" moving average);

5. Fill in a dataframe with signals, for each trading date and contract maturity, attributing value one where

$$EWMA[P|S_k] > EWMA[P|L_k] \tag{2.5}$$

and value zero when the above condition is not met. In this dataframe, a signal 1 indicates a long position recommendation, while a signal 0 indicates a short position, based on the market trend.

In this study, the momentum strategy will be calculated from the yield of the contracts. Through this method, the model was conceptually built to be responsive to changes in the expected rates at a future maturity, rather than changes in contract prices.

2.3 Performance Measurement

For further performance calculation of the deployed trading strategy, a set of measures have been selected based on each one's specific characteristics.

1. Excess Returns

   As in Cogneau and Hübner (2009), excess returns are contrasted with gain measures, in the dimension of value creation. An excess return represents the result of a "zero-investment strategy". So, it represents the payoff from a unit of investment financed by borrowing. In this paper, the benchmark risk-free rate is the Brazilian CDI, the interbank rate in Brazil.

2. Volatility

   The volatility of historical returns account for their standard deviation (σ) for a given time frame and frequency, as in Benninga and Czaczkes (2014). The time frame commonly used varies wildly: Some practitioners use a short term of, say, 30 days, while others use a much longer (up to 1 year) time frame. Similarly, the frequency of returns can be daily, weekly, or sometimes monthly.

   The higher the volatility in outcomes, the higher will be the average value of these squared deviations. Therefore, variance and standard deviation provide one measure of the uncertainty of outcomes, according to Bodie et al. (2013).

3. Sharpe Ratio

   The Sharpe Ratio (SR) is the reward-to-volatility measure, first proposed by William Sharpe in Sharpe (1966) and widely used to evaluate the performance of investment managers, as stated by Bodie et al. (2013). Consistent with Sharpe (1994), the Sharpe Ratio (SR) is calculated as:

   $$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma} \tag{2.6}$$

   In this formula, R_p represents the portfolio return, while R_f is the risk-free rate, and σ is the standard deviation of the portfolio's returns.

4. Sortino Ratio

   The Sortino Ratio corresponds to a variant of the Sharpe Ratio, as described by Bodie et al. (2013).

   The use of standard deviation as a measure of risk when the return distribution is non-normal presents two problems: (1) the asymmetry of the distribution suggests we should look at negative outcomes separately; and (2) because an alternative to a risky portfolio is a risk-free investment, we should look at deviations of returns from the risk-free rate rather than from the sample average, that is, at negative excess returns, as in Bodie et al. (2013).

According to Bodie et al. (2013), a risk measure that addresses these issues is the lower partial standard deviation (LPSD) of excess returns, which uses only negative deviations from the risk-free rate (rather than negative deviations from the sample average), squares those deviations to obtain an analog to variance, and then takes the square root to obtain a "left-tail standard deviation." The LPSD is therefore the square root of the average squared deviation, conditional on a negative excess return. The Sortino Ratio consists of the ratio of average excess returns to LPSD (σ_d):

$$Sortino\ Ratio = \frac{R_p - R_f}{\sigma_d} \tag{2.7}$$

5. Maximum Drawdown and Maximum Drawdown to Volatility

A drawdown is defined as the accumulated percentage loss due to a sequence of drops in the price of an investment, according to Leal and Mendes (2005). The maximum loss from a market peak to a market nadir, commonly called the maximum drawdown (MDD), measures how sustained one's losses can be, as follows:

$$MDD = \frac{Trough\ Value - Peak\ Value}{Peak\ Value} \tag{2.8}$$

One adjustment that can be made is to divide the *MDD* by the calculated volatility of the returns. Scaling the MDD with respect to the standard deviation of returns makes the ratio comparable across strategies, as in:

$$MDD - to - Vol = \frac{MDD}{\sigma} \tag{2.9}$$

3 DATA

3.1 Financial Instrument Specifications

The DI rate, or DI-Cetip, is the average rate of interbank transactions carried out through interbank certificates of deposits. Its value is close to the Selic rate, which guides the monetary policy conduction in Brazil. The Brazilian interest rate futures market shows some particularities that distinguish it from most markets worldwide, as reported by Santos and Silva (2015).

In Brazil, the contract traded represents the present value of a virtual bond whose value at expiry is 100,000 BRL. Each DI contract has, as its underlying asset, the DI interest rate compounded until the contract's expiration date, for this purpose defined as the capitalized daily DI rates verified in the period between the trade date and the last trading day, as published by B3 (2020).

The daily profits and losses of a position shall be settled in cash (payment of debits and receipt of credits) on the following trading session. According to B3 (2020), open positions at the end of each trading session, after being transformed into UP, shall be settled according to the day's settlement price. The variation shall be calculated up to the expiration date, in accordance with the following formulas published by B3 (2020):

$$AD_t = [(PA_t - (PA_{t-1} \times FC_t)] \times M \times N \tag{3.1}$$

Where:

ADt = the daily settlement value, in Brazilian Reais, corresponding to day "t";

PAt = the contract settlement price on day "t", for the respective contract month;

PO = the trading price, in UP, calculated as follows, after the transaction has been carried out:

$$PO = \frac{100,000}{\left(1 + \frac{i}{100}\right)^{\frac{n}{252}}} \tag{3.2}$$

Where:

i = the traded interest rate, expressed as a percentage;

n = number of business days between the trade date and the day preceding the expiration date;

M = the Brazilian Real value of each UP point, as established by B3;

N = the number of contracts;

PAt-1 = the settlement price on day "t–1" for the corresponding contract month;

FCt = the correction factor on day "t", defined by the following formula:

1. When there is one business day between the last trading session and the

day of the variation margin:

$$FC_t = (1 + \frac{DI_{t-1}}{100})^{\frac{1}{252}} \tag{3.3}$$

Where:

DIt-1 = the DI rate, corresponding to the trading session preceding the day to which the variation margin refers, to six decimal places.

2. When there is more than one business day between the last trading session and the day of the variation margin and therefore there is more than one DI Rate published for the period between two consecutive trading sessions, the correction factor will represent the accumulation of all the published DI Rates, as below:

$$FC_t = \prod_{j=1}^{n} (1 + \frac{DI_{t-j}}{100})^{\frac{1}{252}} \tag{3.4}$$

Where:

DIt-j = DI Rate, corresponding to each business day between the previous trading session and the day before the settlement date, to six decimal places.

3.2 Database

The data examined in this study was extracted from the database provided by the Brazilian exchange, "Brasil Bolsa Balcão", available at their website B3 (2020), through a connection established with the AWS server from FinanceHub (accessed August 30, 2020). The extracted database contains the historical series of daily settlement prices, for the different maturities one-day interbank deposit future contracts, each of them denominated a One-day Interbank Deposit Futures (DI) in the local financial market.

The period analyzed ranges from May-10 to May-20, amounting to 2.5k observations. Table 1 shows part of the data obtained for the contracts available at B3, with open interest greater than zero, in the most recent trading date considered for this analysis, which is the 15^{th} of May, 2020.

In sequence, the annualized unitary prices obtained in the data base were converted into annualized rates, which would be used for further calculation in the model, where UP_t is the unitary settlement price of the contract in the date t, that is, the value (in points) corresponding to 100,000 discounted by the interest rate. Bus is the number of business days, in accordance with the Anbima (2020) calendar, between the considered date and the contract's date of expiry, and DI_t is the corresponding rate of the contract that should be calculated in order to construct the accurate yield curve.

The calculated rates for each DI contract available were, then, interpolated in order to build the complete yield curve, comprising all possible maturities. The interpolation method chosen was the spline method, based on its flexibility to adjust to the real data,

	time_stamp	contract	maturity_code	open_interest_close	settlement_price
150470	2020-05-15	DI1	M20	1100495	99875.16
150471	2020-05-15	DI1	N20	4056567	99653.76
150472	2020-05-15	DI1	Q20	336540	99435.65
150473	2020-05-15	DI1	U20	141165	99228.88
150474	2020-05-15	DI1	V20	816069	99026.15
150475	2020-05-15	DI1	X20	111820	98812.06
150476	2020-05-15	DI1	Z20	68885	98609.48
150477	2020-05-15	DI1	F21	3713326	98407.87
150478	2020-05-15	DI1	G21	67320	98168.26
150479	2020-05-15	DI1	H21	52957	97962.94
150480	2020-05-15	DI1	J21	605997	97698.97
150481	2020-05-15	DI1	K21	24395	97438.01
150482	2020-05-15	DI1	N21	980907	96840.57
150483	2020-05-15	DI1	V21	471595	95730.62
150484	2020-05-15	DI1	F22	2290014	94528.89
150485	2020-05-15	DI1	J22	371860	93171.64
150486	2020-05-15	DI1	N22	431745	91748.21
150487	2020-05-15	DI1	V22	53155	90091.32
150488	2020-05-15	DI1	F23	1646384	88589.54
150489	2020-05-15	DI1	J23	27455	86761.08
150490	2020-05-15	DI1	N23	821765	84971.49
150491	2020-05-15	DI1	V23	27485	83104.87
150492	2020-05-15	DI1	F24	593805	81255.15
150493	2020-05-15	DI1	J24	9860	79491.87
150494	2020-05-15	DI1	N24	39654	77582.91
150495	2020-05-15	DI1	V24	20840	75787.89
150496	2020-05-15	DI1	F25	848903	74064.02
150497	2020-05-15	DI1	J25	4170	72399.04
150498	2020-05-15	DI1	N25	11145	70817.43
150499	2020-05-15	DI1	F26	71438	67163.40
150500	2020-05-15	DI1	N26	9425	64040.56
150501	2020-05-15	DI1	F27	388313	60752.14
150502	2020-05-15	DI1	F28	25475	54699.27
150503	2020-05-15	DI1	F29	201417	49684.97
150505	2020-05-15	DI1	F31	31253	40824.11

Table 1 – DIs contracts by maturity and price, with open interest greater than zero, in 15^{th} of May, 2020.

mentioned by McKinley and Levine (1998). A 2-order polynomial was chosen to implement the spline, since many of the lines had few data to be interpolated, making it difficult to use higher order polynomials.

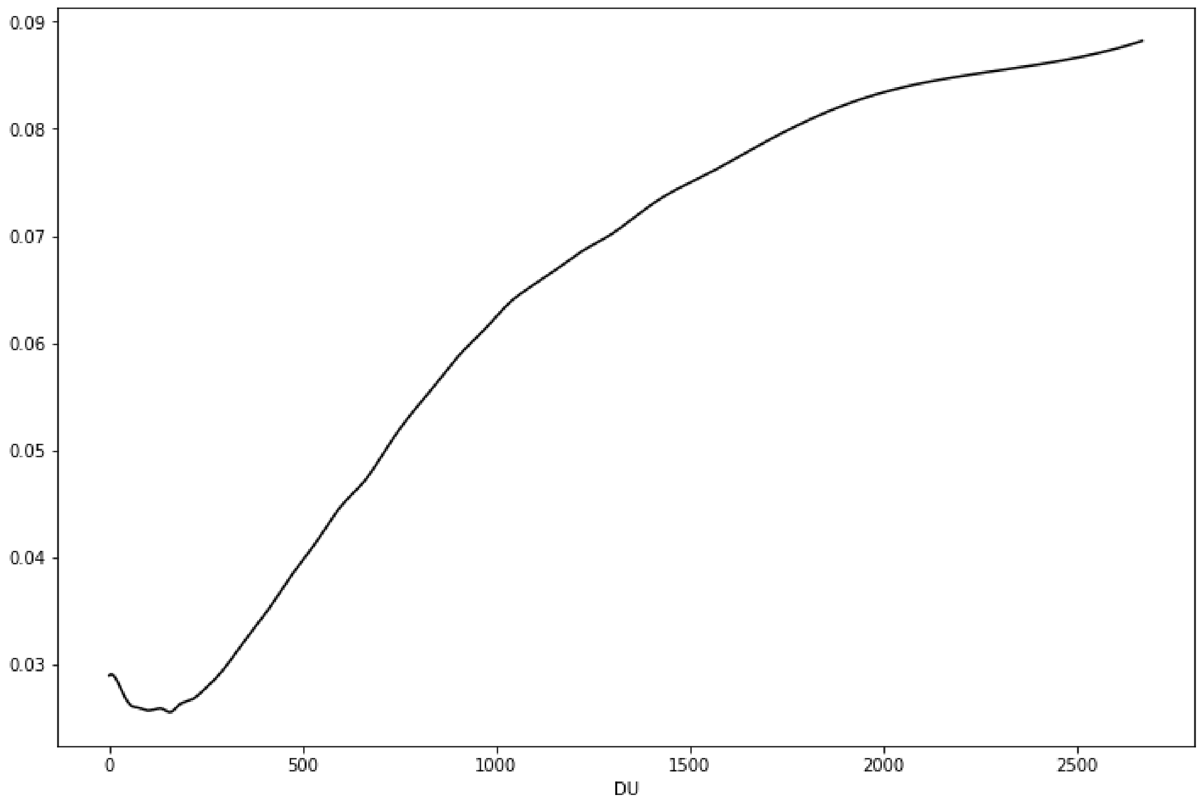

Figure 2 – Interpolated Brazilian Yield Curve at 15/05/2020

The interpolation was smoother in the most recent periods, for which there is a large volume of observations covering all vertices of the curve, both long and short. On the other hand, the method can present disadvantages in the initial periods, where there are fewer observations for longer vertices. In this way, the values are interpolated following the trend of the short curve, which can generate an explosive trend for interest rates in the long run - as can be seen in the high rates of rates for very long terms. In addition, there is no parameter in the model for the level of long-term interest, to which the curve should converge.

4 METHODOLOGY

4.1 Model Structure

Three codes were developed in Python, and the complete model is available online (DARIN, 2020). Both models follow algorithms based on inputs from the finance literature, as described below.

The work starts with data processing. Firstly, the database is obtained from B3, with prices and contracts information on each of the available contracts at any date in the ten year time-series. Then, the DIs unitary prices are converted into yields, and these are interpolated in order to build the complete yield curve for each date in the series.

The database is then modelled according to three different algorithms. To all of the models datasets, there have been applied a few filters. There were only included traded vertices with a minimum of 150 daily open interest and maturity date in January, in order to only select the most liquid contracts. After being subjected to these filters, the resulting DI contracts on 2020 May 15^{th} represented on Table 2. In the model, each DI contract is represented by k, ordered from the shortest to the longest maturity-.

k order	Maturity	Contract	Code
k=1	2022-01-03	DI1	F22
k=2	2023-01-02	DI1	F23
k=3	2024-01-02	DI1	F24
k=4	2025-01-02	DI1	F25
k=5	2026-01-02	DI1	F26
k=6	2027-01-04	DI1	F27
k=7	2028-01-03	DI1	F28
k=8	2029-01-02	DI1	F29

Table 2 – DIs contracts after open interest and liquidity filters, as of 2020 May 15^{th}

The present model analyzes the latest 10 years of the Brazilian yield curve market. This timeline has been selected after analysing the evolution of this specific market in recent periods, seeking for a long horizon through which the market characteristics are consistent. Although B3 provides slightly older data, the market presented comparably lower liquidity and availability of tradable contracts.

For instance, each of the strategies generates signals in a daily basis, being traded on closing prices. No costs have been taken into account, in order to obtain an analysis of the premium in the curve solely.7

4.1.1 Combined Strategies Algorithm

Both models, Carry and Momentum, are combined in this paper into a single strategy.

In this model, the Momentum strategy is introduced as an enhancement. When, in a given date, the momentum signal equals one (payer signal), the model assumes levered long exposure to the contract, whose level should be calibrated according to the following volatility algorithm. Otherwise, when the signal equals zero (receiver signal), the momentum long exposure is neutralized and the model trades only based on carry premium.

Hence, the carry model will account for 0.5 of the total portfolio risk exposure through the entire time series (static strategy):

$$Carry\ Exposure = 0.5 \tag{4.1}$$

When introducing the momentum exposure in the model, which would modify the premium accrual strategy, there were two options of sizing. The first one, would be to open the momentum position with the same size for both strategies: carry and momentum. This way, when the momentum signal prescribed a long position, each of the strategies (carry and momentum) would account for 0.5 of the total portfolio risk exposure. It is important to highlight that attributing equal weights to both strategies would assume a constant risk across all market scenarios.

For this reason, an alternative has been proposed in order to calibrate exposure according to market volatility. The proposed alternative is idiosyncratic, and it attributes, for each contract, the volatility on day t is calculated as the annualized standard deviation (σ) for the returns in a given time frame. Two volatility time frames are considered for this measure; the annual (long time frame) volatility, which comprises the latest 252 trading days observations, and the monthly (short time frame) volatility, which considers the 21 most recent observations. All volatility calculations assume the same daily frequency of observations.

Once the two annualized volatility levels have been calculated, they can be used for sizing the total long exposure when the momentum signal equals 1, as follows:

$$Momentum\ Exposure = \frac{Vol_{252d}}{Vol_{21d}} * 0.5 \tag{4.2}$$

The model assumes the one-year volatility as the pattern for the specific contract. When the shorter volatility (21 days) level is greater than the longer level (252 days), the net effect will be a reduction of the total risk exposure (Momentum Exposure < 0.5). The opposite case also verifies; when the longer volatility (252 days) level is greater than the shorter level (21 days), the net effect will be a leverage of the total risk exposure (Momentum Exposure > 0.5).

This risk management procedure ensures that the momentum strategy will achieve bigger exposure during periods when the market is rising consistently and the volatility stays below its average one-year level, also working as a stop loss when market starts to fall abruptly, the short term volatility rises above the long term volatility and the portfolio suffers with losses. The model will backtest and compare both algorithms, one with volatility cap rebalancing and one without it, to assess its advantages.

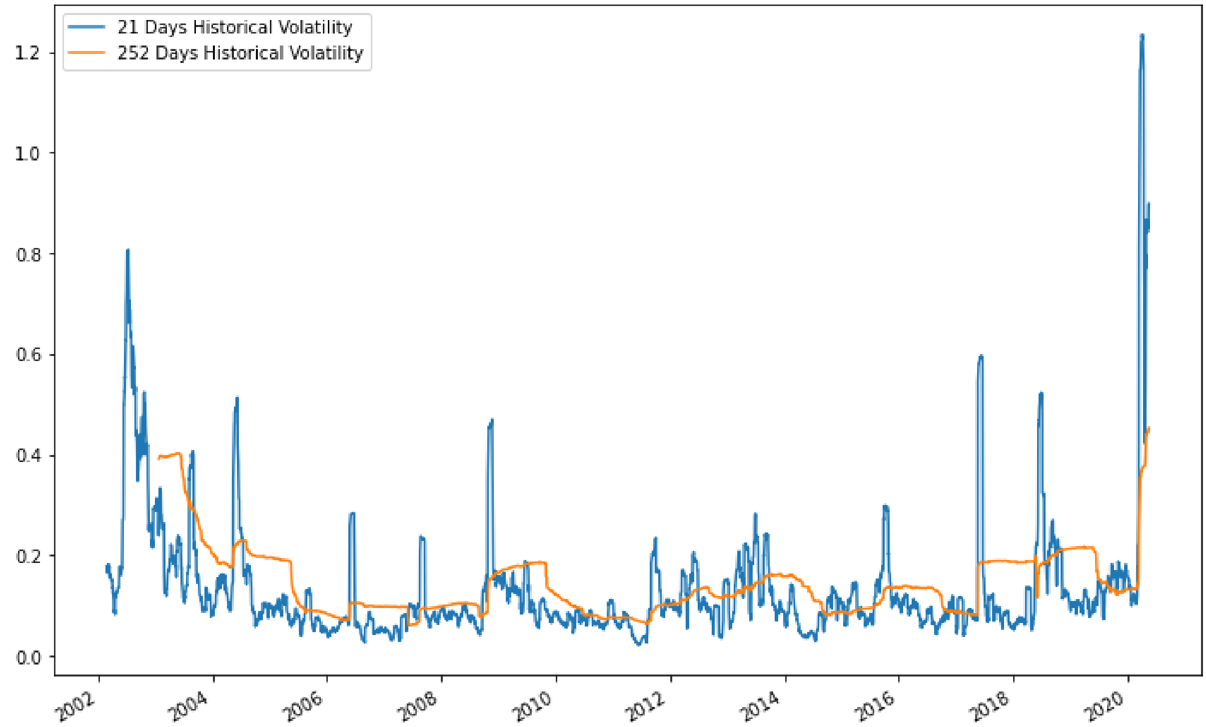

Figure 3 – Annualized Volatility (1 Month, 1 Year): 1 Year Contracts

Figure 3 plots two historical volatility measures, calculated for the 252 business days to maturity yield; the blue line corresponds to the 21 trading days historical volatility, while the orange line corresponds to the 252 business days historical volatility. As the chart indicates, the short-term volatility not only varies considerably more, but it also spikes more in comparison with the long-term measure, supporting the volatility cap control intuition behind algorithm that guides the model.

5 RESULTS

5.1 Backtest and Performance

5.2 Carry Model

The Carry Model yields positive returns throughout the time series, as can be seen in Figure 3. The highest premia concentrates on the longer vertices, in line with finance theory of forward premium. Also, during market crashes, those longer maturities tended to suffer more severe losses, relatively to the shorter maturities contracts.

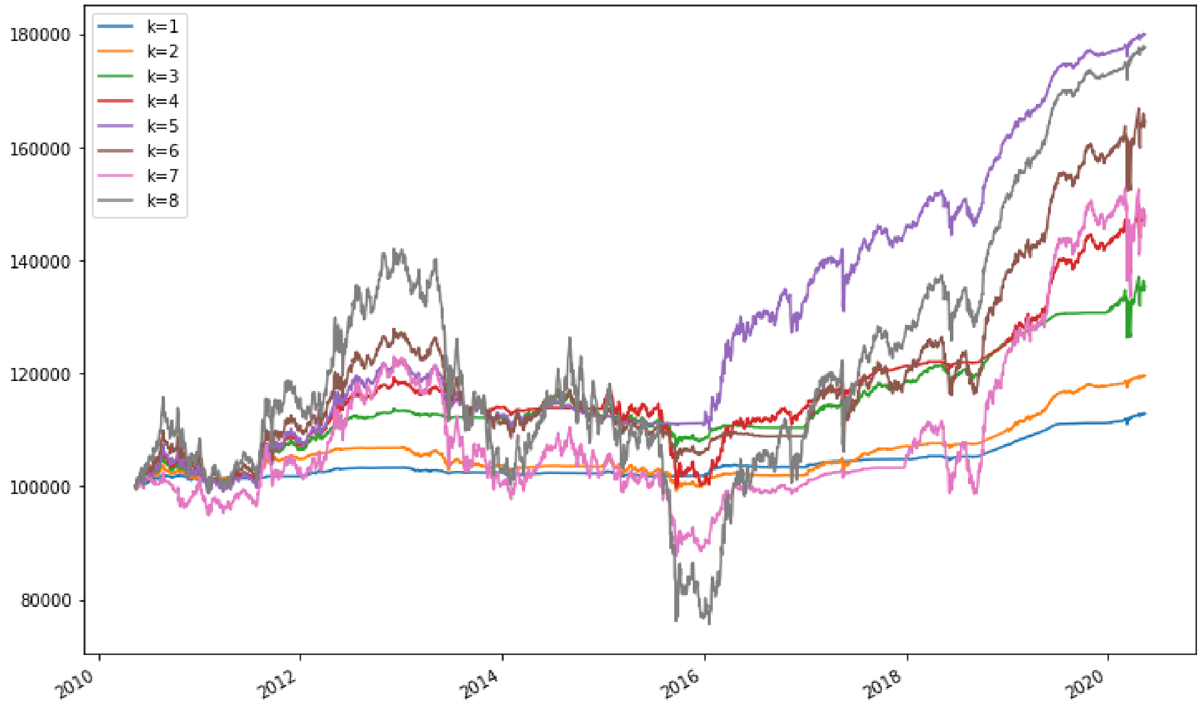

Figure 4 – Carry Model Backtest

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
Frequency	Daily	Daily	Daily	Daily	Daily	Daily	Daily	Daily
Excess ret.	1.25%	1.85%	3.16%	4.09%	6.21%	5.24%	4.03%	6.07%
Vol.	0.75%	1.97%	4.08%	5.36%	5.99%	7.91%	9.87%	15.21%
Sharpe	1.67	0.94	0.77	0.76	1.04	0.66	0.41	0.40
Sortino	1.79	0.97	0.73	0.80	1.15	0.73	0.47	0.47
MaxDD	-1.77%	-7.22%	-8.07%	-16.33%	-10.47%	-18.38%	-28.65%	-46.81%
MaxDD to Vol	-2.36	-3.67	-1.98	-3.04	-1.75	-2.32	-2.90	-3.08
No. Obs	2460	2460	2460	2460	2460	2460	2460	2460

Table 3 – Carry Strategy: Performance Measurement

The carry model presented Sharpe Ratios that ranged from approximately 0.40 (longest maturity contract), to 1.67 (shortest maturity contract), reaching 0.8310 on average. The model also presented, for the longest maturity contract, relatively high volatility (15.2%) and a Maximum DrawDown of 46.8 %. As shows Figure 5, the Sharpe Ratios of the strategy tend to be smaller, the longest the contract maturity.

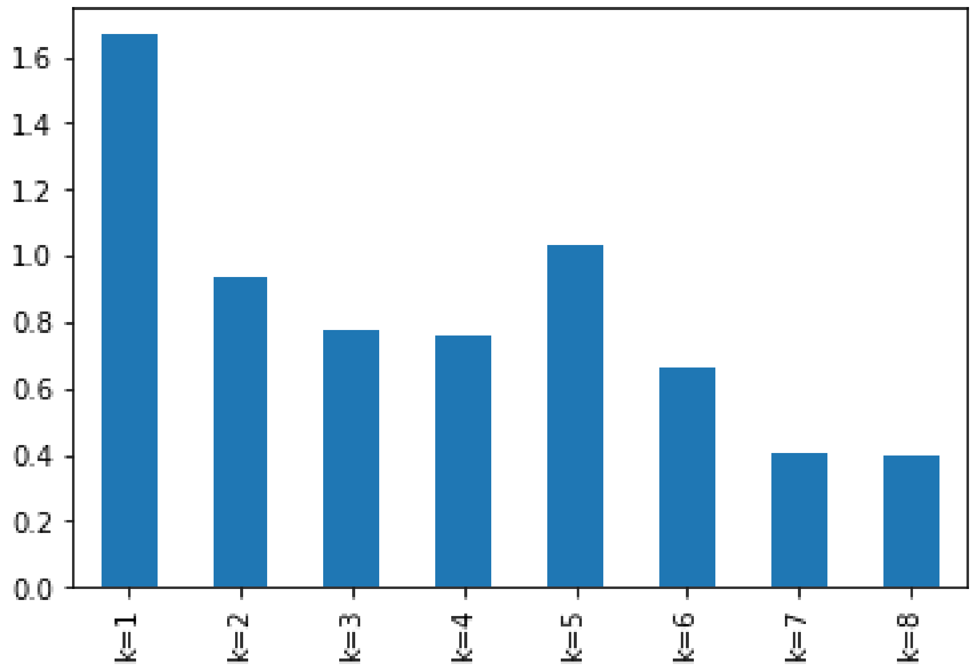

Figure 5 – Carry Strategy Sharpe Ratios

5.3 Momentum Model

The momentum model backtest presented a smoother return profile, where the drawdowns are minimized, when compared to the carry strategy, indicating that the MACD framework succeeded in identifying market trends. However, the Sharpe ratios are considerably smaller on most maturities, indicating more volatility per return in excess of the risk free rate.

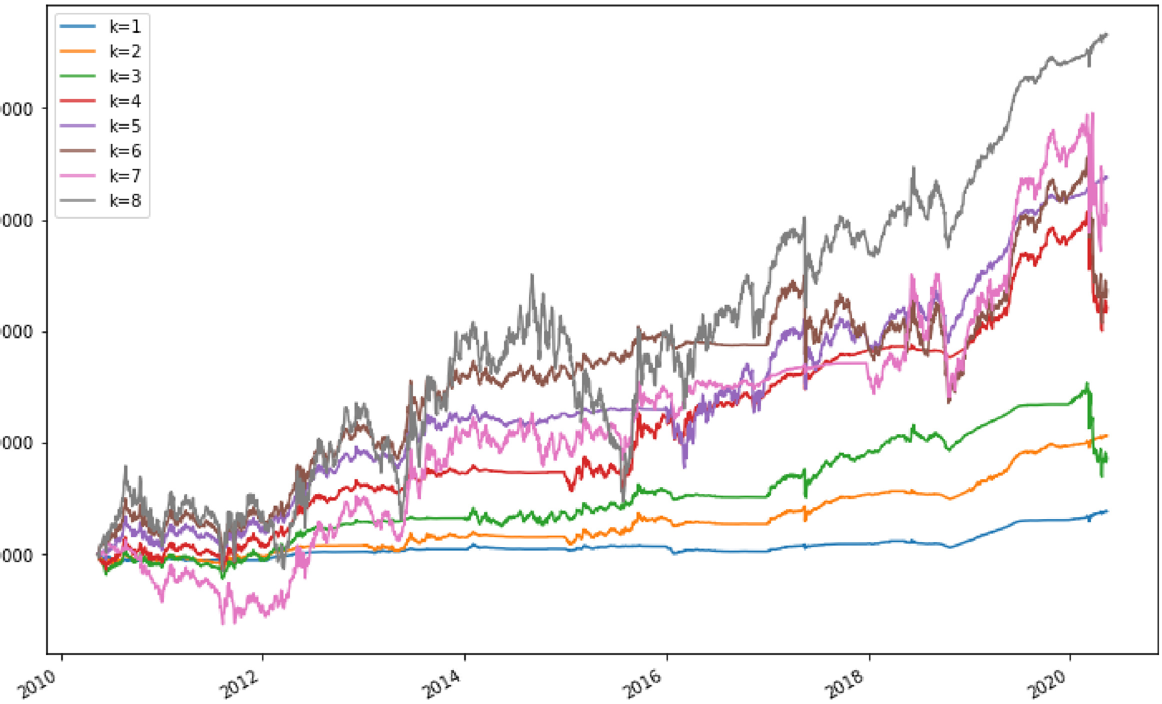

Figure 6 – Momentum Model Backtest (12, 26 MACDs)

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
Frequency	Daily	Daily	Daily	Daily	Daily	Daily	Daily	Daily
Excess ret.	0.76%	1.99%	1.64%	3.83%	5.43%	4.05%	5.04%	6.98%
Vol.	0.75%	1.95%	4.16%	5.25%	6.53%	8.34%	10.00%	12.89%
Sharpe	1.00	1.02	0.39	0.73	0.83	0.49	0.50	0.54
Sortino	1.13	1.15	0.38	0.80	0.91	0.52	0.63	0.69
MaxDD	-1.66%	-2.94%	-12.91%	-13.25%	-9.23%	-17.38%	-14.77%	-27.13%
MaxDD to Vol	-2.20	-1.50	-3.10	-2.53	-1.41	-2.08	-1.48	-2.10
No. Obs	2460	2460	2460	2460	2460	2460	2460	2460

Table 4 – Momentum Strategy: Performance Measurement

As shown in Figure 7, the Sharpe ratios were still greater on the shorter maturity contracts, ranging from 0.39 to 1.02 (0.6884 on average). In comparison with the Carry model, the Momentum strategy performed better for the longer contracts, which tend to be more susceptible to large drawdowns.

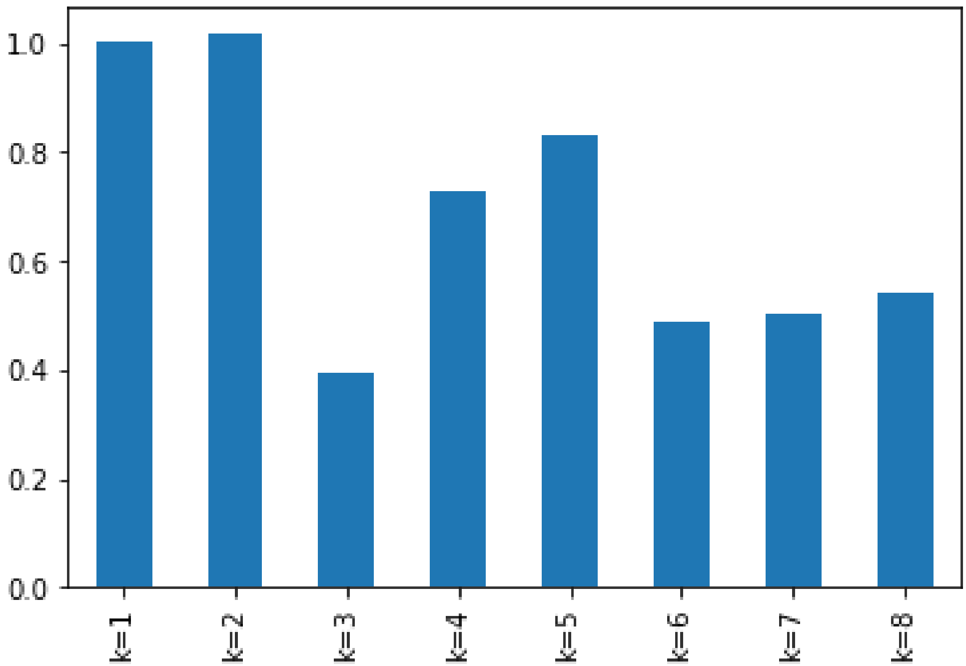

Figure 7 – Momentum Strategy Sharpe Ratios

Figure 8 plots the Selic Target Rate, the target interest rate set by the Brazilian central bank as a monetary policy tool, which influences the level of the short-term interest rates. The vertical lines correspond to dates where the model points to a change of signal, for the 252 business days yield (1 year contracts). As the figure shows, the signal generation is closely related to changes in the Selic rate path, alternating between "buy" and "sell" recommendations accordingly.

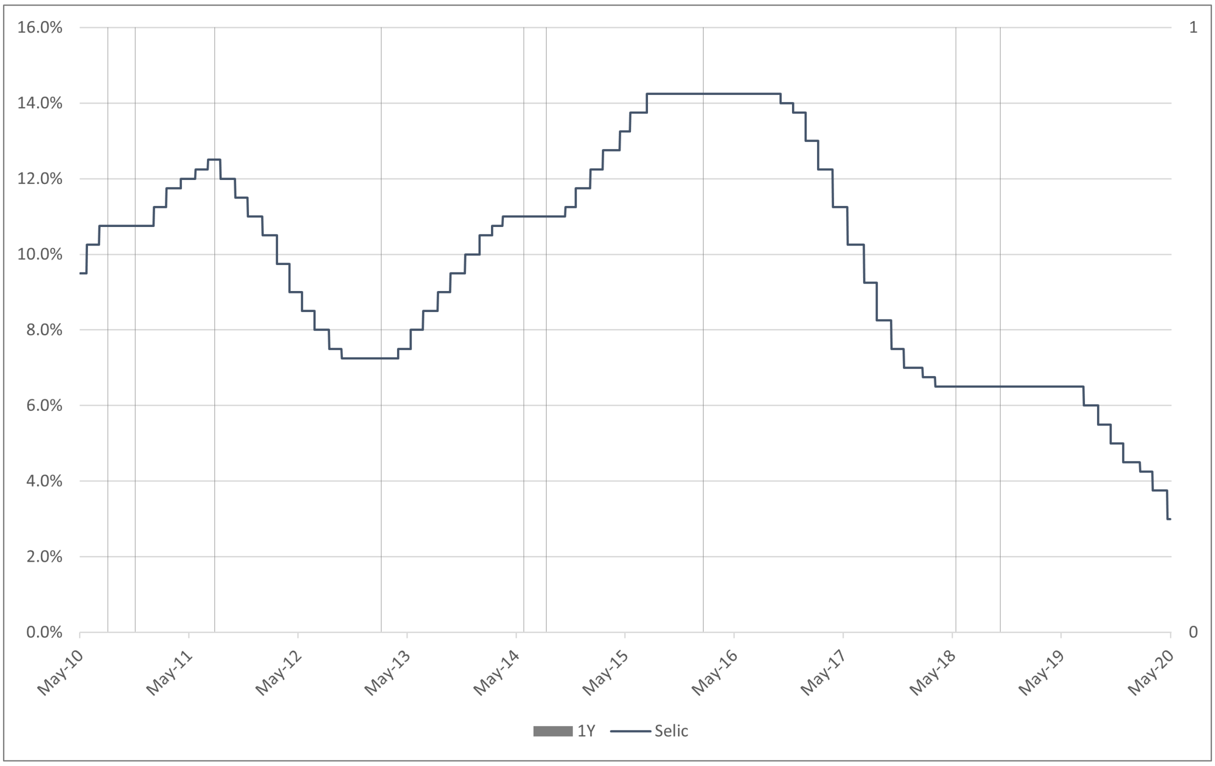

Figure 8 – Brazil Selic Target Rate vs. Change of Signal (1 Year Contracts)

5.4 Combined Strategies Model

When combining the fundamentals of both the Carry and Momentum models, and adding a volatility based rule for sizing positions, the yield trading model not only presents higher excess returns adjusted to volatility, but also produces a trading strategy that is less susceptible to drawdowns. This outcome is due to the combination of a carry accrual strategy with a trend following one, adjusting the position sizing according to the volatility measure. Through this strategy, the model reduces its long exposure during moments of higher uncertainty, and leverages it when market trends are clearer.

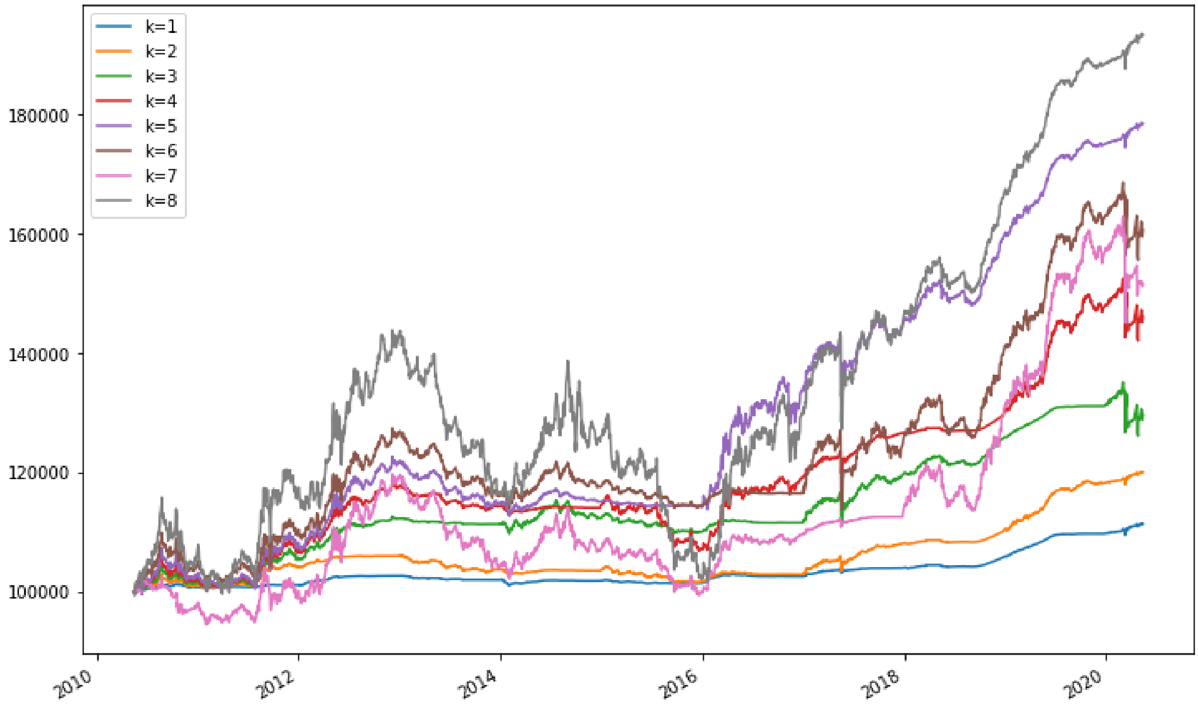

Figure 9 – Combined Strategies Backtest

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
Frequency	Daily	Daily	Daily	Daily	Daily	Daily	Daily	Daily
Excess ret.	1.11%	1.89%	2.69%	3.96%	6.12%	4.98%	4.35%	6.99%
Vol.	0.64%	1.55%	3.63%	4.46%	5.44%	6.86%	7.16%	11.06%
Sharpe	1.73	1.22	0.74	0.89	1.12	0.73	0.61	0.63
Sortino	1.82	1.22	0.66	0.90	1.19	0.73	0.71	0.75
MaxDD	-1.59%	-4.46%	-6.67%	-9.92%	-8.10%	-10.70%	-17.03%	-29.37%
MaxDD to Vol	-2.48	-2.88	-1.84	-2.22	-1.49	-1.56	-2.38	-2.66
No. Obs	2460	2460	2460	2460	2460	2460	2460	2460

Table 5 – Combined Strategy: Performance Measurement (Daily Frequency, With Volatility Cap)

The Sharpe ratios in this model range from 0.61 to 1.73; both the lower and the higher ratios are greater than the ones produced by the two base models. The model also presented an average ratio of 0.9584, consistently superior than the ratios produced by the other models.

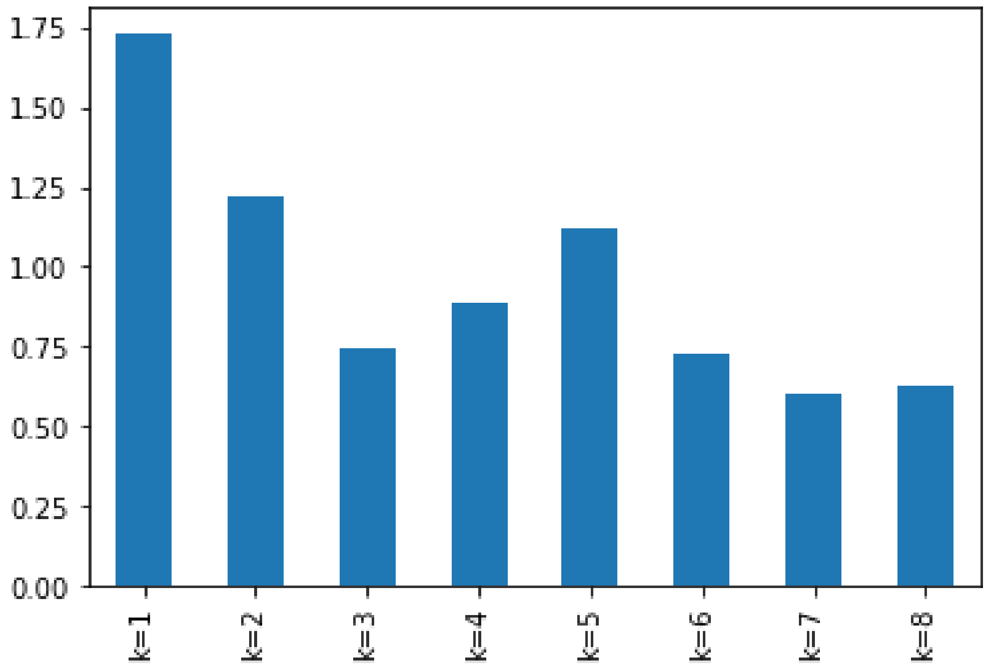

Figure 10 – Combined Strategies Sharpe Ratios

In order to assess the effect of the volatility control, the combined strategies model has been tested without the volatility cap control. As we can see from Table 6, even though the combination produced superior Sharpe indexes relatively to the momentum strategy

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
Frequency	daily	daily	daily	daily	daily	daily	daily	daily
Excess ret.	0.75%	0.91%	1.99%	2.14%	3.57%	2.96%	1.80%	3.12%
Vol.	0.56%	1.56%	2.77%	4.03%	4.30%	6.04%	8.75%	13.01%
Sharpe	1.33	0.58	0.72	0.53	0.83	0.49	0.21	0.24
Sortino	1.43	0.60	0.74	0.57	0.95	0.57	0.22	0.28
MaxDD	-1.65%	-6.81%	-7.21%	-15.10%	-10.24%	-19.41%	-28.10%	-43.79%
MaxDD to Vol	-2.96	-4.36	-2.61	-3.75	-2.38	-3.22	-3.21	-3.37
No. Obs	2460	2460	2460	2460	2460	2460	2460	2460

Table 6 – Combined Strategy: Performance Measurement (Daily Frequency, Without Volatility Cap)

5.4.1 Alternative Frequencies

In order to assess the effect of frequency over the model, two other time frames have been tested. For comparison purposes, the strategy was also backtested in a weekly and a monthly trading frequency.

The purpose behind testing longer time frames in the model lies in the idea of testing if a less responsive algorithm could produce superior results. Additionally, reducing the trading frequency usually implies minimizing the trading costs associated with brokerage, exchange fees and slippage, and consequently verifying the possibility of achieving superior returns. In this model, though, trading costs have not been considered, since the brokerage fees associated with future contracts are usually low, and the model usually suggested holding each of the positions for long time frames.

Nevertheless, the results have been inferior, when compared to the daily frequency model. The longer frequencies turned the models to be less responsive to market changes, subtracting returns and adding volatility, therefore lowering the model's overall Sharpe ratios. Tables 6 and 7 present the results obtained with weekly and monthly frequencies, respectively.

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
Frequency	Weekly	Weekly	Weekly	Weekly	Weekly	Weekly	Weekly	Weekly
Excess ret.	0.23%	0.39%	0.55%	0.80%	1.23%	1.01%	0.88%	1.40%
Vol.	0.29%	0.70%	1.65%	2.03%	2.47%	3.12%	3.25%	5.02%
Sharpe	0.78	0.55	0.33	0.40	0.50	0.32	0.27	0.28
Sortino	0.82	0.55	0.30	0.40	0.53	0.33	0.32	0.33
MaxDD	-1.59%	-4.46%	-6.67%	-9.92%	-8.10%	-10.70%	-17.03%	-29.37%
MaxDD to Vol	-5.47	-6.34	-4.05	-4.89	-3.28	-3.43	-5.24	-5.85
No. Obs	2460	2460	2460	2460	2460	2460	2460	2460

Table 7 – Combined Strategy: Performance Measurement (Weekly Frequency)

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
Frequency	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly
Excess ret.	0.05%	0.09%	0.13%	0.19%	0.28%	0.23%	0.20%	0.32%
Vol.	0.14%	0.34%	0.79%	0.97%	1.19%	1.50%	1.56%	2.41%
Sharpe	0.38	0.26	0.16	0.19	0.24	0.15	0.13	0.13
Sortino	0.40	0.26	0.14	0.19	0.25	0.16	0.15	0.16
MaxDD	-1.59%	-4.46%	-6.67%	-9.92%	-8.10%	-10.70%	-17.03%	-29.37%
MaxDD to Vol	-11.38	-13.21	-8.43	-10.19	-6.82	-7.14	-10.90	-12.17
No. Obs	2460	2460	2460	2460	2460	2460	2460	2460

Table 8 – Combined Strategy: Performance Measurement (Monthly Frequency)

5.5 Additional Findings

5.5.1 Low Volatility Anomaly

Baker, Bradley and Wurgler (2011) have described the "Low Volatility Anomaly", stating and proving cases in which high volatility assets substantially underperformed low volatility assets. The idea behind this concept contradicts the common principle in finance, that associates higher risks with superior returns. The authors highlight that, among the many candidates for the greatest volatility anomaly in finance, a particularly compelling one is the long-term success of low-volatility and low-beta stock portfolios.

In an extension of the literature produced by the referred authors, in this study, a similar pattern could be observed in the Brazilian yield future contracts, as depicted in Figure 11.

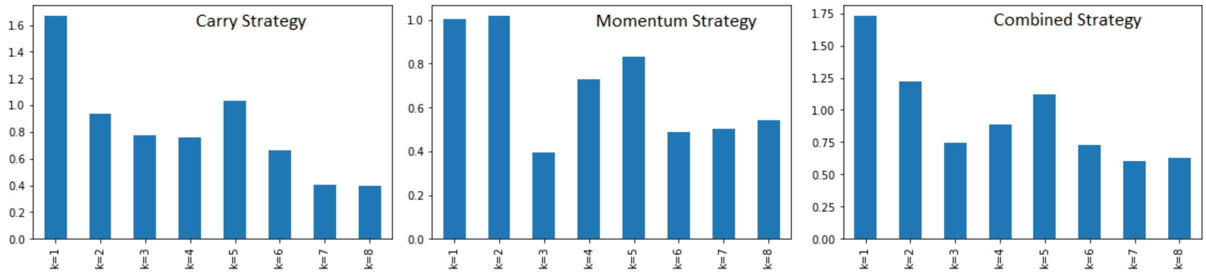

Figure 11 – Sharpe Ratios

The contracts with shorter maturities and lower realized volatility tended to present higher Sharpe ratios. This pattern was consistently found when analyzing both the shortest and longest maturity DI contracts, but could not be precisely observed in the DIs correspondent to the middle of the Brazilian yield curve.

The inverse relation between the superior performance, noted by higher Sharpe ratios, and the volatility level has been observed in all variations of the model. Figure 11 plots the Sharpe ratios obtained from the carry (or long only) strategy, from the momentum strategy (pure trend following), and from the combined strategies model, which have already been presented in previous sections of this study.

6 CONCLUSION

This paper elaborated on the development and backtesting of a trading strategy combining different alternative risks among future yield curve contracts. The objective was to compose a trading strategy capable of enhancing the premium extracted throughout the curve regardless of the yield levels in the country, which have been anchored in this study as the Selic target rate.

When compared to the Brazilian risk-free rate (CDI), and to a pure carry and a pure momentum-based strategy, the proposed model has proven to overperform not only the considered benchmark, but also both of the individual strategies.

The present research contributed to the field of quantitative finance as it aimed to verify if superior returns can be achieved through a combination of carry and momentum strategies, which are strategies based on the expected returns from the slope and curvature of the yield curve, in comparison to the application of each strategy separately or to the benchmark risk-free rate. Moreover, this paper provided information about the application of carry and momentum strategies over the Brazilian yield curve, by means of a time series of exchange traded future contracts, in line with the theory found in previous studies from the financial literature that mainly focus on developed countries.

As the study presents, there is premium in the Brazilian yield curve that can be extracted from alternative factors that do not directly include monetary policy decisions or economic forecasts. Also, as the models show, the combination of a Carry model with a Momentum model has provided superior returns, in comparison to the performance of each of the strategies individually; this exceeding performance has been further enhanced by the introduction of a risk management tool in the model, named a volatility cap. Additionally, the paper results indicate that the Low Volatility Anomaly can be observed among the Brazilian DI contracts, in line with Baker, Bradley and Wurgler (2011) findings.

Much of the knowledge regarding trading strategies often keeps restrained to players in the financial markets industry. Therefore, this field has not been completely explored in the academic scope. The findings in this study should enrich the knowledge on both the fields of trading strategies and of the Brazilian interest rates market, and consequently open opportunities for further academic development in each of these areas.

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